DOI: https://doi.org/10.46354/i3m.2019.sesde.004

POLLUTION ROUTING PROBLEM WITH TIME WINDOW AND SPLIT DELIVERY

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ABSTRACT

In most classic vehicle routing problems, the main goal is to minimise the total travel time or distance while, the green vehicle routing problem, in addition to the stated objectives, also focuses on minimising fuel costs and greenhouse gas emissions, including carbon dioxide emissions. In this research, a new approach in Pollution Routing Problem (PRP) is proposed to minimise the CO_2 emission by investigating vehicle weight fill level in length of each route. The PRP with a homogeneous fleet of vehicles, time windows, considering the possibility of split delivery and constraint of minimum shipment weight that must be on the vehicle in each route is investigated simultaneously. The mathematical model is developed and implemented using a simulated annealing algorithm which is programmed in MATLAB software. The generated results from all experiments demonstrated that the application of the proposed mathematical model led to the reduction in CO_2 emission.

Keywords: vehicle routing, green Vehicle Routing, split delivery, simulated annealing.

1. INTRODUCTION

The Vehicle Routing Problem (VRP) is part of a series of problems that are associated with determining a set of routes in which each vehicle starts moving from a particular warehouse, serving a set of specified customers, and returning to the same warehouse. This problem was first introduced by Dantzig and Ramser (1959) and solved by mathematical methods. In the sequel, Laport et al. (1992) developed a variety of approaches of the branch and bound method to solve the vehicle routing problem. Clarke and Wright (1994) proposed a savings algorithm for solving VRP, which was the basis for many further kinds of researches. With recent advances in solving these problems, and taking into account more complex assumptions and constraints, metaheuristic methods such as genetic algorithm (Potvin and Bengio 1994), Tabu Search algorithm (Ho and Haugland 2004), ant colony optimization algorithms (Reimann et al. 2002), simulated annealing (Saad and Bahadori 2010) and particle swarm optimization (Norouzi et al. 2017) are developed.

In 1990, Split Delivery Vehicle Routing Problem (SDVRP) and its mathematical model were introduced and presented by (Dror and Trudeau 1990) in which the economic aspect of the problem is considered when a customer is served with more than one vehicle. Dror et al. (1994) provided an integer program for the above problem and used the branch and bound algorithm to solve it. The real application of this problem is studied by (Mullaseril et al. 1997) for a food distribution network at a dairy farm in Arizona, USA. In their work, the delivery of goods to the customer is associated with a time limitation and proposed heuristics algorithm by (Dror and Trudeau 1990) is applied. Belfior (2009) applied SDVRP in a case study in Brazil, for a distribution network consisting of a central warehouse and 519 customers in 11 sectors using Neighbourhood Search algorithm. Tavakkoli-Moghaddam and Safaei (2007) developed the Simulated Annealing algorithm for SDVRP with the heterogeneous fleet.

Transportation has irreparable effects on the environment; Consumption of resources, toxic effects on ecosystems and humans, noise and emissions of greenhouse gases (GHG) and pollutants are examples of these risks. Apart from the aforementioned negative effects, emissions of greenhouse gases and carbon dioxide (CO_2) are directly linked to the health of the community and indirect to the destruction of the ozone layer (Bektaş and Laporte 2011). Most research has taken into account economic goals by minimising the distance, the time required or the number of vehicles needed and has neglected attention to environmental goals. Hence, the Green Vehicle Routing Problem (GVRP) has received the attention of scholars since 2006. In the following, two categories including Green-VRP (G-VRP) and Pollution Routing Problem (PRP) are dominated which have focused on reducing the energy consumption and CO_2 emissions respectively (Lin et al. 2014).

In terms of G-VRP, the following studies can be noted. Kara et al. (2007) modelled the Energy Minimising Vehicle Routing Problem (EMVRP) like the capacitated VRP (CVRP) with a new cost objective function based on the total load and Arc length. However, the details of the formulation of fuel consumption are not provided.

Peng and Wang (2009) modelled the VRP based on fuel consumption by considering just the load of the vehicle. In their objective function, minimising both vehicle travel distance and fuel consumption are targeted. They suggested that in order to have lower fuel consumption, serving the customers with high demand must be prioritised rather than customers with lower demand. A formulation of fuel consumption is done by (Xiao et al. 2012). They added a Fuel Consumption Rate (FCR) as a load-dependent function into a CVRP model and developed a CVRP model with the objective of minimising fuel consumption. In their work, they investigated both the distance travelled and the truckload to determine the fuel costs. Kuo (2010) in addition to the travel distances and load weights, also added the transportation speed to the fuel consumption calculation model in time-dependent VRP. Norouzi et al. (2017) developed a new mathematical model based on time-dependent vehicle routing problem to reduce fuel consumption by using Particle Swarm Optimization (PSO) algorithm.

Among the studies that paid attention to PRP, Maden et al. (2010) considered VRP problem with time windows constraint and proposed and implemented the heuristic algorithm in a case study within the UK which received about a 7% saving in CO_2 emission. Palmer (2007) presented an integrated model for routing and carbon dioxide emissions. He considered the role of speed in reducing carbon dioxide emissions in various congestion scenarios with window time and a reduction of 5% in CO_2 emissions is achieved. However, the effect of the weight of the load was not considered in his problem. Bektaş and Laporte (2011) developed a comprehensive objective function of carbon emissions, driver's cost and fuel consumption within the PRP model with or without time windows. In this work, a minimum speed of 40 km/h is considered as an assumption, which is in contrast to the congestion situations. Continuing this research, Demir et al. (2012) investigated the optimal driving speed and showed that the reduction in CO_2 emissions could occur by changing the speed within the network. In this study, SD-VRP formulation is modified to consider the CO_2 emission and guarantee minimum vehicle weight fill level on board in order to formulate the new PRP optimisation model.

2. MATHEMATICAL MODEL

In this research, a Pollution Routing Problem (PRP) with a homogeneous fleet of vehicles, time window, considering the possibility of split delivery and constraint of minimum shipment weight that must be on the vehicle in each route is investigated simultaneously. In the following, an integer linear programming model of the problem is described:

2.1. Input parameters

V: Total number of customers; $V=\{0, 1, \ldots, n\}$; Where node 0 corresponds to the depot and the other nodes in this set of vertexes represent the customers.
A: sets of edges; $A=\{(i,j) \mid i, j\} \in V$ and $i \neq j\}$.
K: Number of available vehicles; $K=\{1, \ldots, k\}$.
Q_k= Capacity of k^{th} vehicle ($k \in K$).
D_i= Customers Demand ($i \in V$).
d_{ij}= Length of edge between the nodes i and j; ; $(i,j) \in A$ and $d_{ij}= d_{ji}$
M_{sk} = Minimum Shipment weight that must be on the k^{th} vehicle in length of each route
At_{ik}= Arrival time of k^{th} vehicle to i^{th} customer
$t1_i$= Order time of i^{th} customer
$t2_i$= Order due time of i^{th} customer
C_{ijk}= CO_2 emission of moving k^{th} vehicle ($k \in K$) between the nodes i and j
Where
$C_{ijk} = \left(\left(TW_k + W_{ijk} \right) \times E_k \right) \times d_{ij}$
Where
TW_k =Tare Weight of k^{th} vehicle
W_{ijk}= Weight of shipments on board of k^{th} vehicle between the nodes i and j
E_k= CO_2 emission rate of k^{th} vehicle

2.2. Decision variables

$$x_{ijk} = \begin{cases} 1 & \text{if } j^{th} \text{ customer is served by } k^{th} vehicle \\ & \text{after } i^{th} \text{ customer} \\ 0 & otherwise \end{cases}$$

y_{ik}= the quantity of the demand of i^{th} customer which is delivered by the k^{th} vehicle.

2.3. Formulation

Therefore, the vehicle routing problem formulation by Dror et al. (1994) can be modified in order to consider the CO_2 emission and guarantee minimum vehicle weight fill level on board in order to formulate the proposed Green Vehicle optimisation model in this study.

The objective function represents minimisation of the total CO_2 emissions generated by the transportation fleet can be written as follows:

$$Min \sum_{i=0}^{n} \sum_{j}^{n} \sum_{k=1}^{K} C_{ijk} x_{ijk} \ , i \neq j \quad (1)$$

The ten model constraints considered in the proposed model are:

a) Constraint represented in equation (2) ensures that each customer is visited at least once which guarantees the possibility of a split delivery.

$$\sum_{i=0}^{n} \sum_{k=1}^{K} x_{ijk} \geq 1, \ j = 1, \ldots, n, \quad (2)$$

b) Equation (3) is about entrance and exit flows (p), where if a node i is visited by vehicle k, then the amount of product from vehicle k that enters and leaves that node must equal the demand at that node. Conversely, if node i is not visited by vehicle k, then the amount of product from vehicle k that enters and leaves that node must be 0. In fact, this constraint guarantee that any vehicle enters each node will definitely leave it.

$$\sum_{i=0}^{n} x_{ipK} - \sum_{j=0}^{n} x_{pjK} = 0\ , p = \ 0, \ldots, n;\ k = 1, \ldots, K, \quad (3)$$

c) Equation (4) guarantees that vehicle cannot continue to serve more customers in length of each route if the weight of its shipment on board, coming down is from a specified minimum shipment weight.

$$\sum_{i=0}^{n} W_{ijk} \geq M_{sk}\ ,\ \ j = 1, \ldots, n; \quad k = 1, \ldots, K \quad (4)$$

d) Equation (5) ensures that the i^{th} customer's demand is completed if at least one vehicle passes through it.

$$y_{ik} \leq \sum_{j=0}^{n} x_{ijk}\ ,\ \ i = 1, \ldots, n;\ k = 1, \ldots, K \quad (5)$$

e) Constraint displayed by equation (6) indicates that all customers demand is entirely fulfilled.

$$\sum_{k=1}^{K} y_{ik} = D_i, \quad i = 1, \ldots, n \quad (6)$$

f) Equation (7) imposes that the loading process on any route should not exceed the capacity of the vehicle.

$$\sum_{i=1}^{n} y_{ik} \leq Q, \qquad k = 1, \ldots, K \quad (7)$$

g) Constraint demonstrated by equation (8) presents the sub tour elimination constraints where (S) refers to any collection of customers having at least *2* and at most *n-1* members.

$$\sum\nolimits_{i,j \in S} x_{ijk} \leq |S| - 1\ \ (S \subset \{1, \ldots, n\});\ |S| \geq 2 \quad (8)$$

h) Equation (9) enforces that all customers are serviced no later than their order due time.

$$t1_i \leq At_{ik} \leq t2_i, i = 1, \ldots, n; k = 1, \ldots, K \quad (9)$$

i) Equation (10) guarantees the decision variables x_{ijk} to be binary.

$$x_{ijk} \in \{0,1\}, i = 0, \ldots, n\ ;\ j = 0, \ldots, n; k = 1, \ldots, K \ (10)$$

j) Equation (11) guarantees that the decision variable yik is positive.

$$y_{ik} \geq 0, i = 1, \ldots, n;\ k = 1, \ldots, K \quad (11)$$

3. SIMULATED ANNEALING

Simulated annealing algorithm is an effective meta-heuristic optimization algorithm for solving optimization problems which is presented by (Kirkpatrick et al., 1983) and adapted from the Metropolis–Hastings algorithm (Metropolis et al., 1953). They proposed a gradual freezing technique to solve the hard optimisation problems. The main advantage of the simulated annealing algorithm is its ability to do not remain at the optimal local point and move to the global optimum point.

In generic term, the algorithm consists of two loops: the first loop reduces the temperature from the initial temperature to the final temperature and the second loop identifies the number of repetitions at each temperature. The factors affecting the timing of temperature reduction include the initial temperature, the final temperature, how to reduce the temperature and the number of repetitions in each temperature.

Simulated annealing algorithm starts from an initial solution and then in a repeat loop it moves to neighbouring solutions. If the neighbour's solution is better than the current one, the algorithm puts it as the current solution. Otherwise, the algorithm accepts that solution with the probability of *exp* ($-\Delta E$ / T) as the current solution. In this regard, ΔE is the difference between the objective function of the current solution and the neighbour's solution and T is a parameter called temperature. At each temperature, several repetitions are performed, and then the temperature is slowly reduced. In the initial steps, the temperature is set very high, so it is more likely to accept worse solutions. With the gradual decrease of temperature, in the final steps, there will be fewer probabilities for accepting worse solutions, and so the algorithm converges to a good solution. Figure 1 illustrates the general structure of the simulated annealing algorithm.

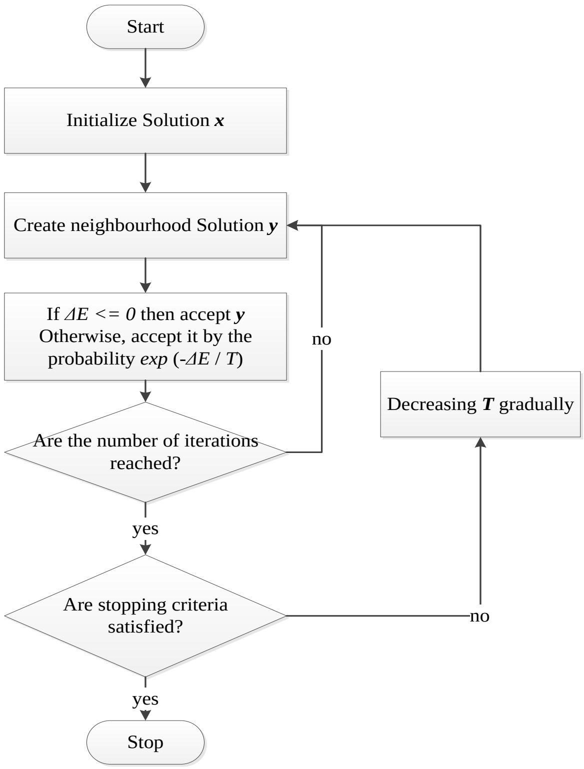


Figure 1: Simulated annealing algorithm flow chart

4. COMPUTATIONAL RESULTS

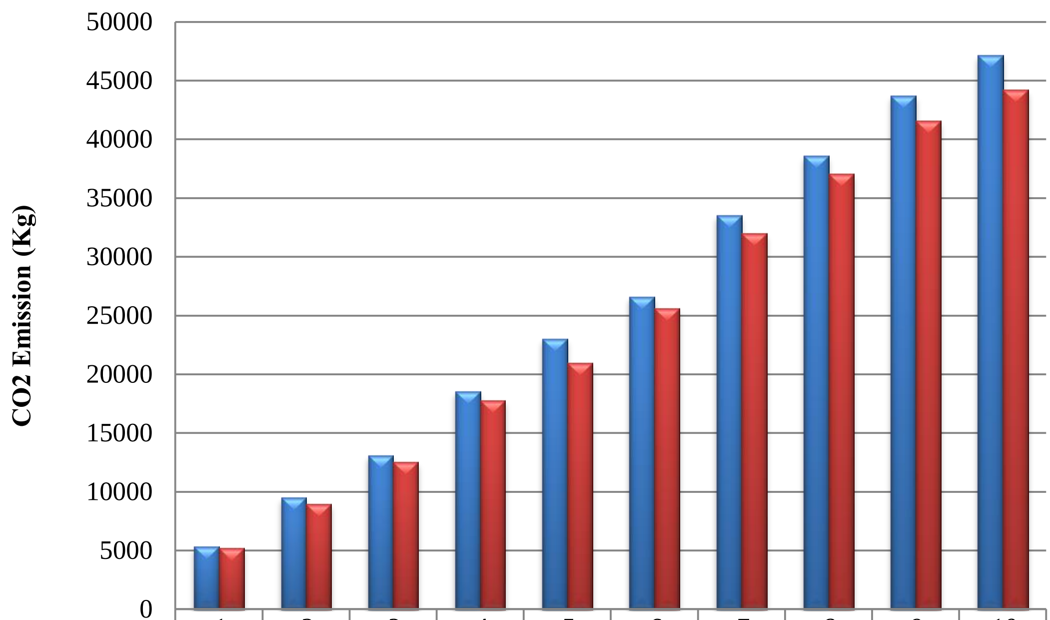
To evaluate the efficiency of the proposed mathematical method, numerical experiments with a single central depot and customer numbers of 20, 40, 60, 80, 100, 120, 140, 160, 180 and 200 are generated randomly; this will lead to ten different experiments. Customer's locations are randomly generated in the space of up to 200 in the *x* and *y* directions and depot's location is concentrated in the centre of this space randomly. Customers' demands weights (kg) are randomly selected from minimum 500 to maximum 3500 Kg to allow split delivery as the capacity of the vehicle is determined to be 3000 kilograms with a CO_2 emission rate of 0.0005442 kg per km (Hill et al., 2012). The number of the vehicle is considered equal to the number of customers through each experiment. The customer's order time (the date that order is placed) is determined randomly to be between January 1th, 2019 and January 7th, 2019.

Order due time of each customer is specified to be the maximum three days. It means customer's order must be filled within three days after that its order is placed.

In order to evaluate the efficiency and verify the proposed model, it was performed using simulation annealing through two approaches; one by considering the minimum weight of shipments on board (*Ms*) (equation 4) and one without its consideration. Comparison of the results shows in all experiments, by applying the *Ms* (Minimum Shipment weight) constraint, the obtained outputs from the proposed model are improved in terms of both mileage and CO_2 emission (see Table 1). In Figures 2 and 3, the quality of the generated results through the ten experiments is shown in the improvement of the two criteria of CO_2 emission and mileage.

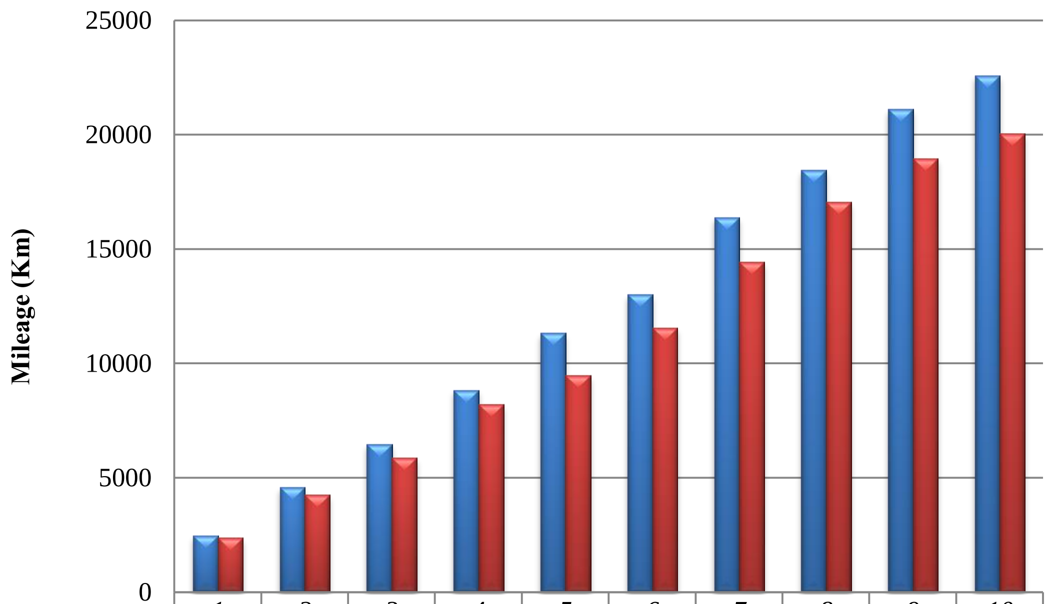
Table 1: The obtained reduction rate of mileage and CO_2 emission by applying the *Ms* (constraint represented by equation 4)

Experiment Number	**Mileage reduction (%)**	**CO_2 emission reduction (%)**
1	4.2	2.1
2	7.2	5.7
3	9.0	4.4
4	7.1	4.1
5	16.3	9.0
6	11.2	3.6
7	12.0	4.6
8	7.6	4.0
9	10.2	4.9
10	11.2	6.2



	1	2	3	4	5	6	7	8	9	10
Without Ms	5341	9515	13120	18557	23030	26588	33554	38606	43724	47169
With Ms	5227	8974	12542	17803	20962	25628	32012	37081	41586	44221

Figure 2: Comparison of the generated results in terms of the CO_2 emission criterion



	1	2	3	4	5	6	7	8	9	10
Without Ms	2486	4592	6469	8848	11342	13022	16400	18461	21117	22599
With Ms	2382	4263	5889	8221	9492	11564	14435	17064	18968	20060

Figure 3: Comparison of the generated results in terms of the Mileage criterion

5. CONCLUSION

In this paper, a new approach for Pollution Routing Problem (PRP) with time window and the possibility of split delivery was proposed to minimise the CO_2 emission by investigating minimum shipment weight that must be on the vehicle in length of each route.

The proposed mathematical model was developed and implemented using a simulated annealing algorithm in MATLAB software.

The results demonstrated that the application of the proposed model has led to a reduction in CO_2 emission.

In relation to future work, the proposed PRP will be developed with heterogeneous fleet and limit the number of vehicle for further evaluation of its effectiveness.

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