32nd European Modeling & Simulation Symposium
17th International Multidisciplinary Modeling & Simulation Multiconference

ISSN 2724-0029 ISBN 978-88-85741-44-7 © 2020 The Authors.
DOI: 10.46354/i3m.2020.emss.004

Digital algorithm for determining the root-mean-square signal

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Abstract

The need to determine the root-mean-square values of alternating signals often arises during the circuit simulation of electronic devices. In this paper, there is introduced a digital algorithm for the direct estimation (measurement) of the root-mean-square value of deterministic and random signals of arbitrary shape for the current signal sampling over the set time interval. It requires the minimum number of simple arithmetic operations while generating the result and ensures a high degree of estimation accuracy. Simulation is then carried out demonstrating the high efficiency of the proposed algorithm. There are analyzed the characteristics of the resulting estimate within a wide frequency range of the measured signals. It is shown that the algorithm can be software-implemented and then it will be a part of an application package, and it also can be hardware-implemented and then one uses the microprocessor system or the field programmable gate arrays.

Keywords: Alternating signal; root-mean-square value; measurement algorithm; fast digital processing; simulation

1. Introduction

The root-mean-square (RMS) value of the signal (current or voltage) is an energy estimate of its level and is determined as the square root of its power. If the processed signal is a centered random process, then its RMS value is equal to the signal dispersion (Kasatkin and Nemtsov, 1986; Poularikas, 2000; Sklar, 2017). Estimating (measuring) the RMS signal value is a common task in various areas of electronic engineering including circuit simulation. For example, simulators of measuring devices (multimeters) are the parts of the software packages such as MultiSIM (Herniter, 2003) and TINA-TI (Texas Instruments, 2008).

In the general case, determining the RMS value of a signal requires integrating its square at the set interval corresponding to the repetition signal period. If the signal shape and parameters are known, then the desired RMS value is calculated by the well-known formulas. If the signal has a complex (non-harmonic) shape and its period is changing or unknown, then the computational procedure is more difficult to implement in practice. In this case, the development of the algorithm for determining the RMS value of a signal is only then relevant, if it involves a minimum number of simple arithmetic operations and provides the required measurement accuracy.

2. Calculating the Root-Mean-Square Signal Value

The power of the periodic signal $s(t)$ is determined by the expression Kasatkin and Nemtsov, 1986; Poularikas, 2000)

$$P = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} s^2(t)\, dt \quad (1)$$

where T_0 is the signal period, and t_0 is any arbitrary reference time of integration upon which the value of the integral (1) does not depend.

The RMS value of the periodic signal (current or voltage) $s(t)$ of an arbitrary shape is determined by the expression (Northrop 2005, Bird 2007)

$$S_{RMS} = \sqrt{\frac{1}{T_0}\int_{t_0}^{t_0+T_0} s^2(t)\, dt} \quad (2)$$

In order to calculate the value (2), it is necessary to know the signal period, but that is not always realizable, especially, when the signal frequency changes during measurements.

For an arbitrary integration interval T, firstly, one defines the value

$$\tilde{S}_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} s^2(t)\, dt} \quad (3)$$

It coincides with (1) when T is a multiple of T_0 and, in the general case, depends upon T and t_0. The value $\tilde{S}_{RMS}$ can be considered as the estimate of the RMS value of a signal and its calculation does not require the knowledge of T_0.

If the harmonic signal

$$s(t) = S\cos(\omega t + \phi) \quad (4)$$

is processed, then from (3) one gets

$$\tilde{S}_{RMS} = S_{RMS}\sqrt{1 + \frac{1}{\omega T}\cos(2\omega t_0 + 2\phi + \omega T)\sin(\omega T)} \quad (5)$$

where $S_{RMS} = S/\sqrt{2}$ is the exact RMS value of the periodic signal (Kasatkin and Nemtsov, 1986). In (4), the notations are: S is the amplitude, $\omega = 2\pi/T_0$ is the frequency, φ is the initial phase of the signal. As the product of trigonometric functions in (5) is not greater than unity in absolute value, the relative error of the estimate (5) of the integral (3) is determined by the inequality

$$\delta = \left|\frac{\tilde{S}_{RMS} - S_{RMS}}{S_{RMS}}\right| \le \delta\frac{1}{4\pi K_{max}} \quad (6)$$

where $K = [T/T_0]$ is the number of signal periods within the integration interval, $[\cdot]$ is an integer part.

In Figure 1a, there is shown the dependence of δ_{max} (6) upon the normalized integration time T/T_0. In particular, it follows that this error is less than 0.8% under $K = 10$ already. In Figure 1b, one can see how the error of the harmonic RMS value estimate (5) changes under $\phi = 0$ and $t_0 = 0$. Similar results also hold for other signal parameter values. Thus, the estimate (3) of the RMS value does not require knowledge of the signal period and provides a sufficiently high accuracy when $K > 10 \div 20$.

If N signal samples s_i following at the interval Δt are available that implies that $T = N\Delta t$, then the integral (3) can be calculated by means of the method of rectangles (Korn and Korn, 2000) as follows

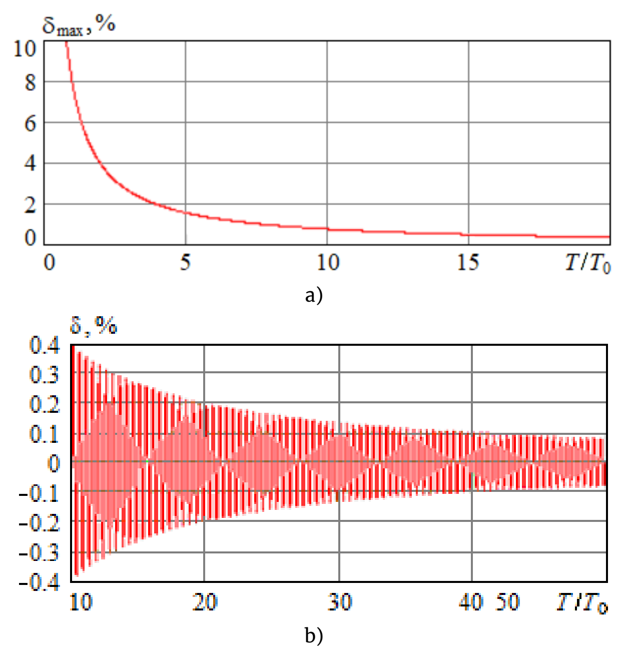


Figure 1. The error of the standard estimate of the harmonic root-mean-square value

$$\tilde{S}_{RMS_i} = \sqrt{\frac{1}{N}\sum_{k=0}^{N-1} s_{i-k}^2} \quad (7)$$

It should be noted that numerical integration methods (Korn and Korn, 2000) require the generation of $K_0 = 50 \div 200$ samples over the signal period. Thus, when measuring the RMS value of a signal, it is necessary to take $N = K_0 K >> 1000$ samples from the output of the analog-to-digital converter; and the measurement accuracy will increase with N. Therefore, in order to effectively implement the estimate (7), a fast computational procedure should be used with the minimum number of arithmetic operations. It is proposed to apply such a procedure that is based on the general approach of fast digital signal processing described by Chernoyarov et al (2018, 2019).

3. The Algorithm for Measuring the Root-Mean-Square Value of a Signal

3.1. The Structure of the Algorithm for Measuring the Root-Mean-Square Value of a Signal

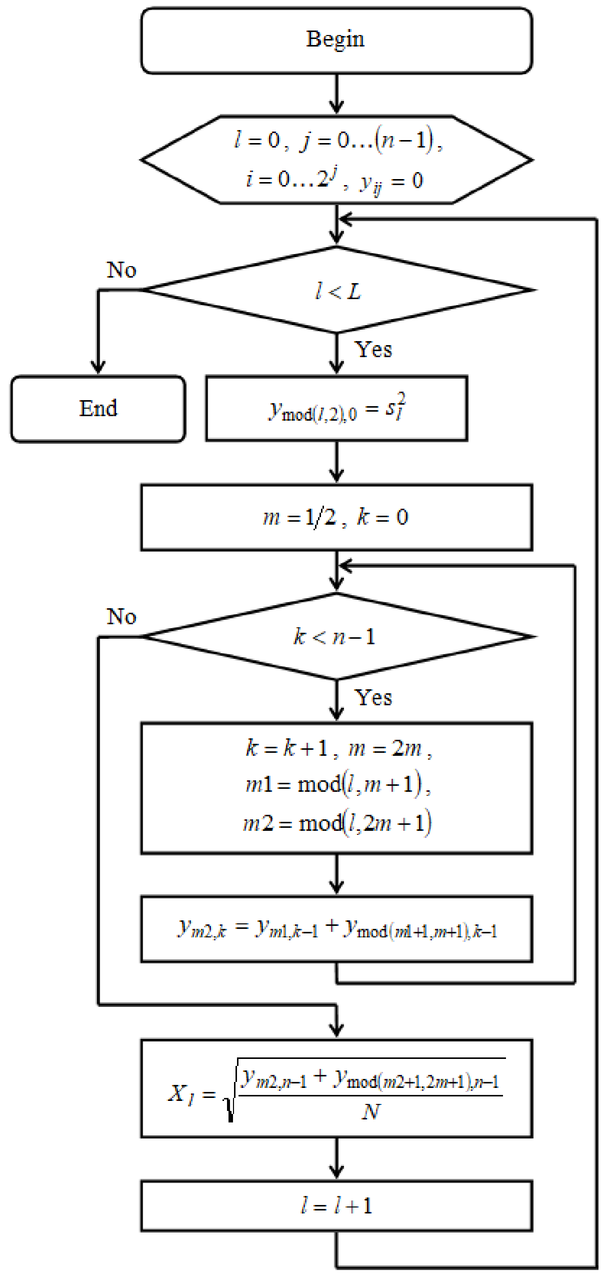
In Figure 2a, there is presented the block diagram of the algorithm (procedure) for determining the RMS signal value using received samples. Here the operation $mod(a, b)$ means calculating the module of the integer number a in terms of the base b. One cane see that this algorithm includes processing of the general sequence formed by L samples of the signal s_l, and, as a result, the RMS signal value is then determined for N current samples ($N = 2^n >> 1$). When initializing the measurement algorithm, the zero values of y_{ij} are set in the memory cells $j = \overline{0, (n-1)}$, $i = \overline{0, 2^j}$, whose number is equal to $2^n + n - 1$ or $N + n - 1$.

Within the processing cycle, each sample is squared and then, during n accumulation cycles, where $n = log_2 N$, the sums are calculated of 2, 4, 8, ... squares of the neighboring samples in (7), and the RMS signal value is then determined. Under $N = 2^{10} = 1024$ there are required $n = 10$ operations of summation, while under $N = 2^{16} = 65536$ – $n = 16$ such operations, respectively. It should be noted that when generating the addresses of memory cells, the integer values are calculated modulo $2^k + 1$ in addition to summing. It leads to additional costs.

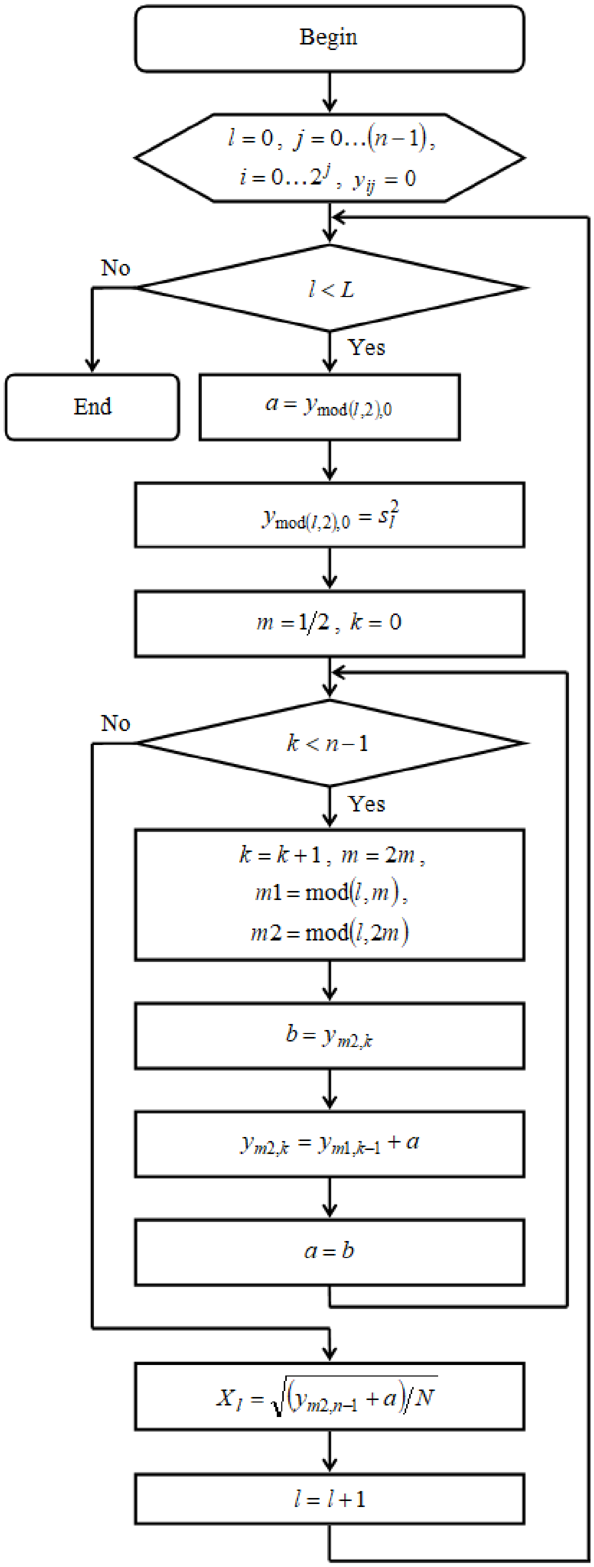
In Figure 2b, the algorithm for determining the RMS signal value is presented, in which the operations of calculating integers modulo 2^k are used. It can be easily implemented by means of a binary mask.

The square root calculation is performed by means of standard algorithms, i.e., using the power series (Chernoyarov and Goloborodko, 2008) or the Heron formula (Korn and Korn, 2000), for example.

As it can be seen, the unit quantity of calculating the RMS value for the signal sample is minimal and does not depend on the total sample size. A hardware implementation of the considered algorithm is also possible.



a)



b)

Figure 2. The block diagram of the algorithm for determining the RMS signal value

3.2. The Harmonic Root-Mean-Square Value

The analysis of the measurement accuracy of the harmonic (4) RMS value using the estimate (7) is carried out by means of simulation. In Figure 3a, the dependence is plotted of the normalized RMS value $\tilde{S}_{RMS_i}/S$ upon the current normalized time $i = t/\Delta$ (where $\Delta = 1/f_S$ is the sampling interval). It is assumed that the signal frequency is $f_0 = \omega/2\pi = 10$ kHz (the signal period is $T_0 = 1/f_0 = 100$ μs), the sampling frequency is $f_S = 1$ MHz ($\Delta = 1$ μs), the sample size is $N = 4096$, the number of samples within the period is $K_0 = 100$, and the number of periods within the averaging interval is $K \approx 41$. At the initial stage, the shifters are filled during 4.096 ms, and then the current measurements are conducted, and they are, as one can see, fairly accurate. The right normalized result is equal to $1/\sqrt{2}$, and it is drawn by dashed line.

In Figure 3b, the error is shown of the measurement results (hundredths of a percent). Their fluctuations are caused by sample shifting during the realization of the harmonic signal.

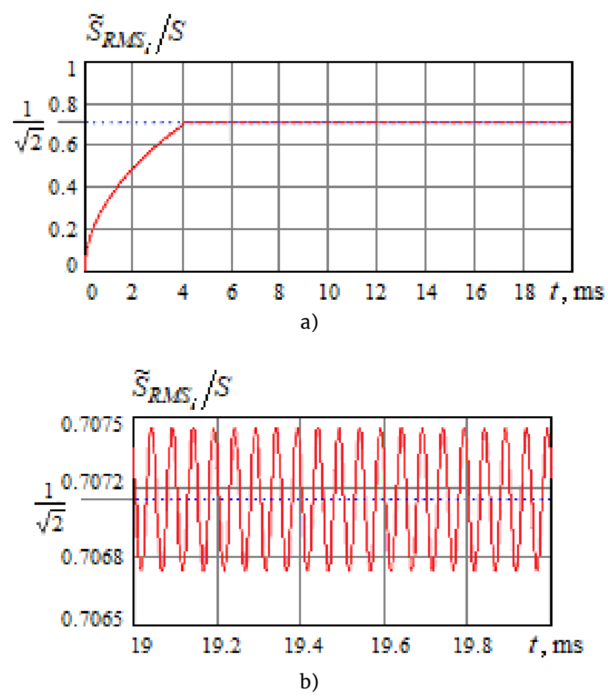


Figure 3. The results of measuring the normalized harmonic RMS value

3.3. The Root-Mean-Square Value of the Sawtooth Signal

Now let us suppose that the sawtooth signal with the period T_0 is being processed. The reference realization of such a signal is drawn in Figure 4.

In Figures 5a and 5b, there are presented the results of measuring the RMS signal value $\tilde{S}_{RMS_i}/S$ (where i is the number of the current sample) in both general and steady states, respectively, while, according to (1), the exact RMS value is equal to $S_{RMS} = 2.877$. It is assumed that $N = 4096$ (100 samples are formed within the signal period). As it can be seen, in that example the high measurement accuracy is also provided.

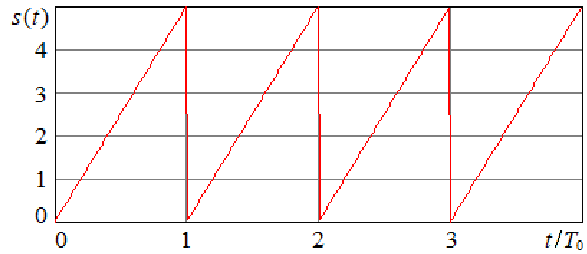


Figure 4. The example of the sawtooth signal

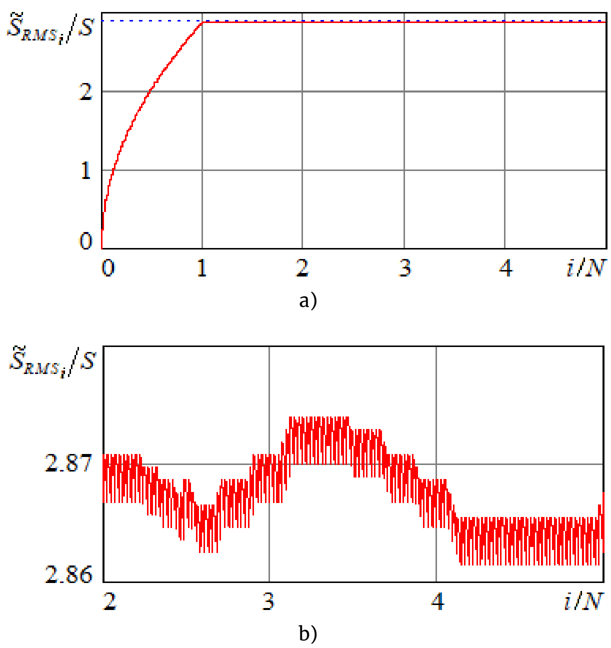


Figure 5. The results of measuring the normalized RMS value of the sawtooth signal

Thus, the introduced algorithm can be effectively used to determine the RMS value of a signal of an arbitrary shape, and it does not require time synchronization.

4. Measuring the Root-Mean-Square Value of a Noise

The algorithm presented in Figure 2 allows us to measure the RMS value of a random signal (noise). In Figure 6, there is shown the realization of the samples s_i of the band Gaussian random process with zero mathematical expectation and dispersion (mean power) S^2.

In Figure 7a, there is drawn the dependence of the measured normalized value $\tilde{S}_{RMS_i}/S$ upon the number i of the processed sample, and in Figure 7b one can see the same dependence but for $i > N$.

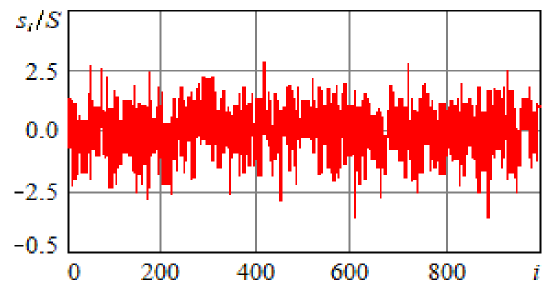


Figure 6. The realization of the centered band Gaussian random process

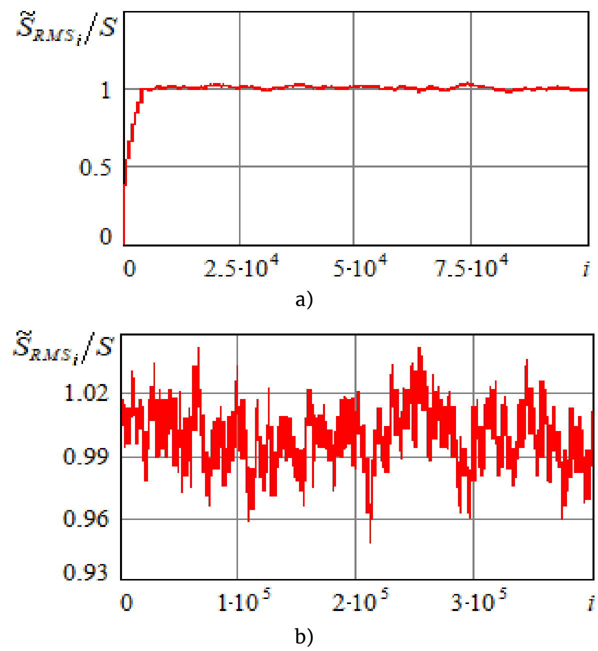


Figure 7. The results of measuring the normalized RMS value of the band Gaussian random process

Under $N = 4096$, the RMS relative measurement error is equal to 1.4%. And if $N = 1024$, then it increases up to 2.4%, while if $N = 65536$, then it decreases down to 0.44%.

In Figures 8, 9, there are presented the similar simulation results for the band random process characterized by the lognormal distribution (Crow and Shimizu, 1988) with the parameters $\mu = 0$ and $S^2 = 1$. In Figure 8, one can see the type realization of such a process and in Figure 9 – the dependences of the measured normalized value $\tilde{S}_{RMS_i}/S$ upon the number i of the processed sample under $N = 4096$. Here the dotted line corresponds to the exact value $S_{RMS} = e = 2.718$ while the RMS relative measurement error is equal to 0.85 %.

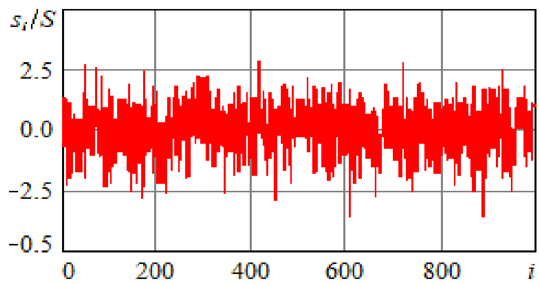
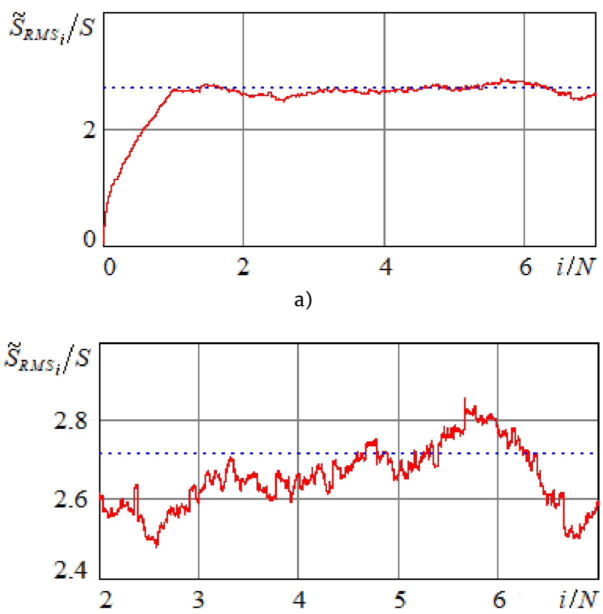


Figure 8. The realization of the band lognormal random process



b)

Figure 9. The results of measuring the normalized RMS value of the band lognormal random process

It can be noted that in order to determine the mean square of a random process (that matches the dispersion or mean power if the process is centered), in the algorithm presented in Figure 2, it is not necessary to extract the square root, and that simplifies the calculation.

5. Conclusions

Application of the considered algorithm for determining the RMS signal value makes it possible to implement high-speed simulation models of voltmeters and ammeters (multimeters), and their readings will not depend upon the waveform. These algorithms do not require both the knowledge of the signal period and the time synchronization. It is shown that a sufficiently high accuracy of the direct RMS value measurement can be achieved while processing both deterministic and random signals of arbitrary shape. The measurement error decreases rapidly with the processed signal sampling size increasing, while the computational cost increases proportionally to the logarithm of this value.

Funding

This study was financially supported by the Ministry of Science and Higher Education of the Russian Federation (research project No. FSWF-2020-0022) as well as RFBR and CNRS (research project No. 20-51-15001).

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