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Extending the Modelling Approaches for Dynamic Hybrid Systems to Accommodate Contact Model

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Abstract

This paper deals with modelling dynamic hybrid systems. Aim is to introduce a modelling approach which incorporates a modelling environment for contact models. This incorporation can be achieved by introducing so-called moving guard sets for the model definition of dynamic hybrid systems. Based on the setup of a dynamic hybrid system using hybrid automata, the mathematical model description will be extended accordingly. A short discussion of the model extension presenting possible applications. Furthermore, we will demonstrate the extended approach and its applicability for dynamic contact models in form of a case study using pendulum systems. The contribution will present a possibility to bring DHS closer to entity based simulation models, which describe contact models on the basis of an individual set of rules for each involved entity. In the conclusion and outlook, further steps and investigations to classify this modelling approach will be presented.

Keywords: dynamic hybrid systems, coupled systems, contact models, pendulum systems

1. Introduction to Dynamic Hybrid Systems

This paper uses hybrid automata as a framework for dynamical hybrid systems (DHS), as defined in Branicky (2005) and Körner (2014). One key component of the definition of the DHS in this framework is the definition of guard sets within the state space X and the corresponding jump mapping to perform the discrete event.

In the classic setup, the guard region is a pre-defined subset $G \subset X$ located in the state space X. The setup for an exemplary 2 to 1 dimensional transition is illustrated in Figure 1. When the trajectory enters the guard region the corresponding jump mapping is activated and hence influences the dynamic of the hybrid system.

The idea of this modelling approach is based on knowing the location of the guard region in each state space. This guard region needs to be predefined and is fixed by the modelling description.

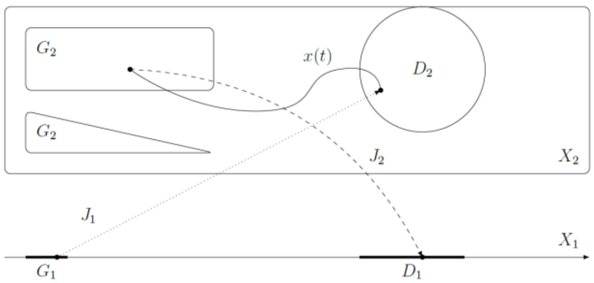

Figure 1. DHS event transition with guard regions G_1, G_2 and jump mappings J_1, J_2 from 1- to 2-dimensional state spaces.

1.1. Mathematical Formalism of Hybrid Automata as Framework for DHS

This section introduces the mathematical framework of dynamic hybrid systems by using hybrid automata to enable a framework extension including moving guards.

Based on Branicky (1994,2005) as well as Liberzon (2003), the mathematical definition of dynamic hybrid system and its components of the DHS environment are given by:

- Finite set of discrete states L.
- Continuous state space X, typically $X \subset \mathbb{R}$.
- Finite set of symbols A, which labels the edges.
- Continuous communication space $W = \mathbb{R}^q$.
- Finite set of events, called the transitions, defined by $(l, a, Guard_{l,l'}, Jump_{l,l'}, l')$, where $l, l' \in L; a \in A$; $Guard_{l,l'} \subset X$; $Jump_{l,l'} \subset XxX$.
- $Inv: L \to \mathcal{P}(X)$, the location invariant where the trajectory of a particular state $l \in L$ is located.
- $Act: L \to F_L$, the mapping the dynamic of a particular state $l \in L$ by assigning $F_l(x, \dot{x}, w) = 0, x \in X, w \in W$.

The tuple $(L, X, A, W, E, Inv, Act)$ is called a hybrid automaton and defines the mathematical framework to characterize a specific DHS. Referring to Figure 1, the guard set is represented, on the one hand, by $G_1 \subset \mathbb{R}$ and on the other hand by set $G_2 \subset \mathbb{R}^2$ which is not connected. The sets D_1 and D_2 are the so-called destination sets, describing the value set of the jump maps with repect to the state vector after executing the transition. As a graphical illustration for such a hybrid automata, a directed graph, as illustrated for a particular setup in Figure 2, can be used.

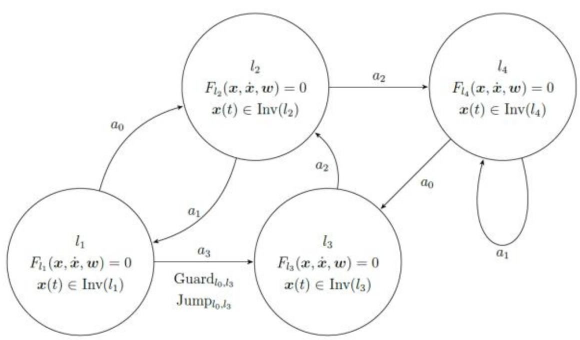

Figure 2. Directed graph as an illustration for the mathematical framework of a DHS.

The example, depicted in Figure 2, assumes the location invariants in the same state space.

A trajectory of such DHS is the connection of the trajectories of each involved dynamical system

$$F_l(x, \dot{x}, w) = 0 \tag{1}$$

which are connected by the jump mappings, as given in Figure 3. Such a connection of individual trajectories of dynamical systems, given as jump mapping, is called a hybrid trajectory. If the location invariants are in the same state space the hybrid trajectory appears as an interrupted curve. The definition still makes sense, even in case of different state spaces in each location.

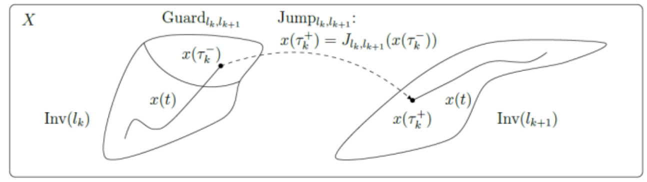

Figure 3. Illustration of $Guard_{l,l'}$ and , $Jump_{l,l'}$ in a state space X.

1.2. Examples of DHS

In this section, typical examples showing the kind of systems addressed by this modelling approach Will be presented.

The first and most cited example is the bouncing ball. A ball with a certain mass is ideally bouncing in a vertical dimension. The height is quantified by $s = s(t)$, the velocity $v = v(t)$ and acceleration is leading to the dynamic system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ -g \end{pmatrix}, \tag{2}$$

where g denotes the gravitational constant and the state vector x is given by $x(t) = (s, v)^T$. The hybrid character is given by modelling the bounce, motivated in Branicky (2005). When the bounce occurs, the bouncing process loses part of its energy. If t^* denotes the event time and $c \in (0,1)$ models the loss of energy, the jump mapping is given by

$$x(t^*{}_+) = -cx(t^*{}_-). \tag{3}$$

The corresponding hybrid automaton is illustrated in Figure 4.

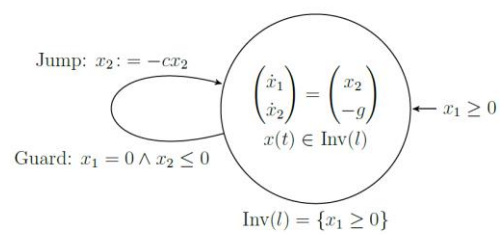

Figure 4. Directed graph illustrates the hybrid automaton of the bouncing ball.

Another famous group of examples are pendulum systems. These systems consist of various pendulum combinations, defined by its mechanical relations. Either the pendulums are combined in a sequence, meaning that the second pendulum is connected to the mass of the first one and so on. Such systems are called double or multiple pendulum systems. Another type are pendulum systems arranged in a row, where collisions are possible. Mechanical sketches of those two types of pendulum systems are depicted in Figure 5.

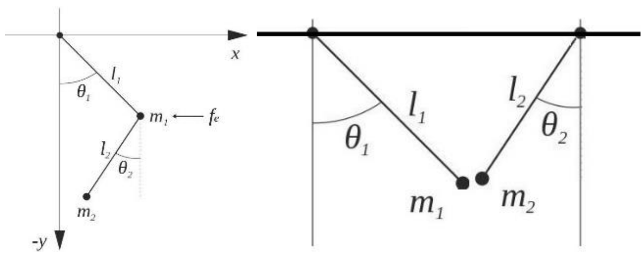

Figure 5. Pendulum systems: Left a double pendulum, right two pendulums in a row.

Further DHS examples are published in Körner, et al. (2010) and Körner, et al. (2016). The different structures of DHS are distinguished by its system properties. Various combinations of parameter changes, state vector changes, a structural changes, etc. are possible. Nevertheless, for each event the definition of the corresponding guard region is necessary.

2. Framework Extension to moving Guard Regions

In the framework definition, presented in section 1.1, the guard region is a pre-defined set within the state space and enables a particular modification of the model description when the state vector $x \in X$ enters this subset of X.

In several setups, this is a well-founded approach to elaborate the mathematical model definition. Additionally, there are setups where the dynamic of a particular system is built by a simulation model, where each involved component has their own dynamic. The contact behavior of such components is implemented by interactions based on neighborhoods and corresponding laws for handling possible collisions.

A famous and similar model approach is the cellular automaton (CA), e.g. see Wolfram (1984). Each entity of a cellular automaton has a particular law of motion within the cells space (grid) and if two entities are going to interact, they are occurring in each others neighborhoods, a subset of the cell space. If an entity is detected in the neighborhood particular collision rules are applied and the propagation of the entities are persuade.

It is obvious that there are several similarities comparing cellular automata approach with DHS automata. The law of motion of the CA is given as discrete equation systems, while DHS are described with differential equations. In CA the neighborhood is moving with the entity and in the classical DHS setup the guard region is a fixed predefined subset. To include the neighborhood concept in the DHS approach results in the possibility to extend contact model approaches.

The extension of the mathematical framework can be done by adapting the guard region. The definition of section 1.1 needs to be adapted in that way, that $Guard_{l,l'}$ depends on the state vector x_l in order to fulfill

$$Guard_{l,l'}(x_l) \subset Inv(x_l) \subset X. \qquad (4)$$

Due to the fact, that $Guard_{l,l'}(x_l)$ depends on the state vector x_l in a particular state l, this type of guard is called moving guard. An graphical interpretation of such moving guards can be found in Figure 6.

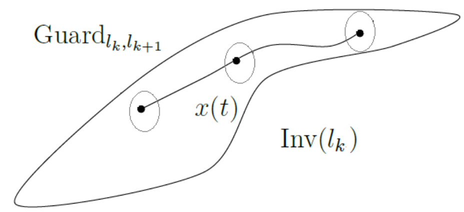

Figure 6. Location invariant $Inv(l_k)$ and three exemplary positions of the moving guard set $Guard(x)_{l,l'}$ along the trajectory of x.

3. Case Study

Two pendulums in a row, as shown in Figure 5 to the right, are chosen to demonstrate the feasibility of the presented framework environment. The moving guard approach allows to split the two pendulums and implement both simulation models separately. The interaction between the two point masses is implemented by the moving guard region.

The basic mathematical model is given as the mathematical pendulum equation

$$\ddot{\varphi} + \frac{g}{l}\sin(\varphi) = 0, \qquad (5)$$

where φ is the angle, and l the length of the rope of one single pendulum. This differential equation is transformed to

$$\begin{aligned} \dot{\varphi} &= \omega, \\ \dot{\omega} &= -\frac{g}{l} sin(\varphi), \end{aligned} \qquad (6)$$

where ω is the angular velocity.

Defining $x(t) = (\varphi(t), \omega(t))^T$ as the state vector, equation (6) defines the state space representation of the dynamic system. For each pendulum, the dynamic system can be implemented and for each of those the trajectory develops independently. The moving guard represents the collision check of the pendulum masses and therefore defines the DHS.

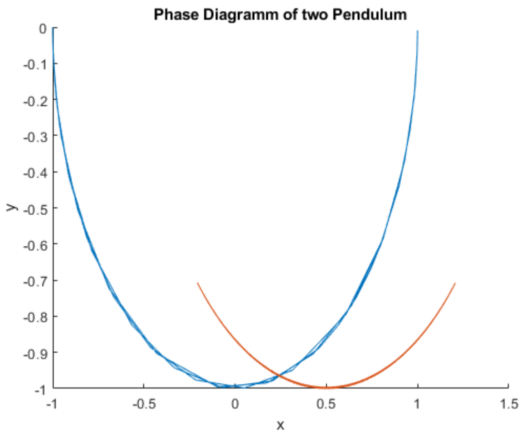

Figure 7. $\mathbb{R}^2$ is the state space for to parallel pendulum systems with initial angles $\theta_{1,0} = \frac{\pi}{2}$ and $\theta_{2,0} = \frac{\pi}{4}$ with same length $l = 1$.

In Figure 7, the trajectory of the two pendulum systems are represented. Both trajectories correspond to different initial values and the intersection of the trajectories indicates the point in the state space, where a event might happen. The occurrence of an event depends on the initial conditions $\theta_{1,0}$ and $\theta_{1,0}$. In this example, similar to Figure 5, the moving guards are assumed as circles with particular radius which effects the tolerance of the possible collision.

4. Summary and Discussion

The framework extension of hybrid automata to include moving guards can be seen as a transfer of the neighborhood aspect of cellular automata into the continuous modelling approach using dynamic systems.

One of the major benefits is, that simple dynamic system descriptions combined with the moving guards enable a more complex DHS behavior of the overall system.

A limiting disadvantage is the fact that for the implementation of the moving guards the fully discretized model description needs to be already known. This means, that the implementation of the moving guards needs to be included in the numerical model of the DHS.

As a consequence, built-in methods, such as ODE or DAE solver, cannot be used anymore. The numerical model needs to be known or derived from the system description to enable implementing this modelling approach.

It is worthy to apply this new modelling approach if simple components can be connected by using the moving guards to the DHS model instead of using the more complex approach defined in section 1.1. There is no general rule to define which case is feasible for which approach. Nevertheless, moving guards enrich the variety of modelling methods.

5. Conclusions and Outlook

The extension of hybrid automata to model DHS brings two different modelling approaches closer together. The moving guards are inspired by the neighborhoods of cellular automata and the dynamic systems as method to model the progress.

Further steps are to investigate more complex case studies applying the moving guards. In addition, traditional examples of cellular automata should be implemented by using DHS with moving guards. Several particle movement models using cellular automata should be reviewed with respect to this process. Benchmarking and comparisons of the two different modeling approaches should be done.

Finally, from the mathematical modelling point of view, it is important to investigate the relation between discrete CA models and continuous DHS models. Then the follow up question would be, if a discretized DHS model with moving guards results in a CA model.

References

Branicky M. S., Borkar V. S. and Mitter S. K. (1994). A unifyed framework for hybrid control. In 33rd IEEE Conf. Dec- Contr., pages 4228-4234.

Branicky, M. S. (2005), Handbook of Networked and Embedded Control Systems, Chapter Introduction to Hybrid Systems, pages 91-116, Birkhäuser Boston.

Liberzon D. (2003), Switched Systems: Stability Analysis and Control Synthesis. Birkhäuser Boston, Boston.

Körner A., Heinzl B., Rößler M. (2010), BCP - A Benchmarik for Hybrid Modelling and State Event Modelling. 7th EUROSIM Congress on Modelling and Simulation, pages 1032-1042.

Körner A. (2014), Approaches for State Event Handling by Simulation Algorithm and via Model Description, 22. Symposium Simulationstechnik, HTW Berlin, pages 219-224.

Körner A., Breitenecker F. (2016), State Events and Structural-dynamic Systems: Definition of ARGESIM Benchmark C21, Simulation Notes Europe, 26(2), pages 117-128.

Wolfram S. (1984), Cellular automata as a model of complexity. Nature, 311:419-424.