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Interaction of ultrashort light pulses with waveguide Bragg gratings: a class of exactly solvable mathematical problems.

Ivan O. Goriachuk[1,*], Ilya M. Asharchuk[1] and Viktor I. Sokolov[1,2]

[1]Institute on Photonic Technologies, Federal Research Center «Crystallography and Photonics», Russian Academy of Sciences, Leninsky ave. 59, Moscow, 119333, Russia
[2]Federal Research Center «Scientific Research Institute for System Analysis», Russian Academy of Sciences, Nakhimovsky ave. 36, b.1, Moscow, 117218, Russia

[*]Corresponding author. Email address: io.gorjachuk@physics.msu.ru

Abstract

The propagation of light pulses within waveguide Bragg gratings is investigated. The exact mathematical solution of the problem for the grating with rectangular groove shape is considered. The solution is valid for arbitrary grating amplitude and length both in the vicinity and far from Bragg wavelength. For this solution the distribution of field amplitudes within the grating, its reflectivity and the structure of resonances are studied. The effects of light pulse broadening, shape transformation and time delay due to interaction with the waveguide Bragg grating are analyzed using numerical simulations.

Keywords: thin films, optical waveguides, Bragg gratings, waveguide modes, ultrashort light pulses.

1. Introduction

Optical waveguide interference filters with the longitudinal phase Bragg grating have extraordinary narrow transmission gap (at the order of 0.1 nm) and permit to separate individual spectral lines. Based on them distributed-feedback lasers gradually displace gas-discharge ones and now are widely used in integrated optical technologies.

The principal of their operation is related with multiple coherent light reflections on each groove of the grating, leading to total reflection of individual (Bragg) wavelength. The period of the grating, the shape of the groove and its amplitude determine the transmission spectrum of the waveguide filter. Due to the big number of grooves (reaching tens of thousands) the device exhibits very narrow reflection resonances, comparable with the spectral width of ultrashort light pulses.

We consider the two dimensional model of the optical waveguide interference filter, admitting the exact analytical solution. We analyze the structure of this solution and calculate the reflectivity and transmittivity spectra of the filter. It is demonstrated that the suggested model allows investigating the processes of ultrashort light pulses scattering on the periodic grating.

2. State of the art

The diffraction of light pulses on periodic structures (Sukhoivanov and Guryev, 2009) even in one spatial dimension (Karpov and Stolyarov, 1993) represents a complicated mathematical problem, often solved numerically. To simplify the task we consider a model of a planar waveguide interference filter with the phase grating, for which the exact mathematical solution of Maxwell equations can be constructed by the help of Bloch theory, widely used for similar periodic problems in quantum mechanics (Kronig and Penney, 1930). In contrast to approximate approaches, such as Kogelnik theory (Kogelnik and Shank, 1972) or the method of sequential approximations, the considered model is exact. It is valid both close and far from the Bragg resonance and for arbitrary amplitude of the phase grating. This model permits us to investigate the structure of resonances, reflectivity and transmittivity of long waveguide Brag grating filters in broad spectral range and for various parameters of the periodic structure. The results are also valid for thin-film interference filters and mirrors.

3. Materials and Methods

3.1. Model description

The suggested model is based on the exact solution of Maxwell`s equations in the self-consistent field regime for the planar waveguide, schematically presented in Figure 1. The waveguide consists of three layers: the light-guiding core film and two claddings. The phase grating, representing the alternating regions of low and high refractive index, is written in the waveguide plane (along the y-axis). The dielectric permittivities εi of the materials in all the three layers of the light-guiding structure vary with equal amplitude Δε. The waveguide under consideration consists of the sequence of alternating short homogenous light-guiding regions of the length $d/2$, where the waveguide mode effective refractive indices remain constant and equal ${\beta_1}^2 = \beta^2 + \Delta\varepsilon$ and ${\beta_2}^2 = \beta^2 + \Delta\varepsilon$, while the mode profile $e(z)$ is the same at every region.

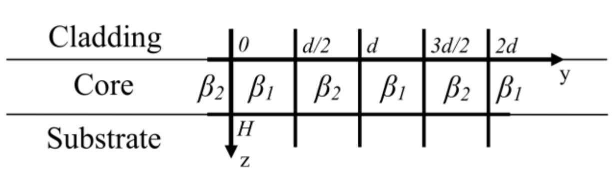

Figure 1. The structure of the planar waveguide with the phase grating written.

Within each homogenous region the complex amplitude of electric field strength in the TE-polarized mode is given by

$$\vec{E}(y,z) = \hat{x}\left(Ae^{i\beta_n ky} + Be^{-i\beta_n ky}\right)e(z) \qquad (1)$$

for $n = 1,2$, where $k = 2\pi/\lambda$, $\hat{x}$ is the unity vector along the x axis and complex coefficients A, B differ from one region to another. On the other hand, the structure under investigation is d-periodic along the y-axis. The propagation of light throw it is described by the second order differential equations with periodically changing coefficients. For these equations the Floquet-Bloch theorem is valid, which states that their solution has the form of the multiplication of a periodic function $\Phi(y)$ and the exponential (throughout the whole considered structure):

$$\vec{E}(y,z) = \hat{x}\Phi(y)e^{i\tilde{\beta}ky}e(z). \qquad (2)$$

3.2. The structure of the solution

To determine the value of the parameter $\tilde{\beta}$ and find the explicit form of the functions $\Phi(y)$ in the equation (2) it is required to equate (1) and (2), impose the sawing conditions on the electric and magnetic field strength at the middle of the period and require the periodicity of the function $\Phi(y)$ by imposing appropriate conditions at the period boundaries. It permits to derive the following dispersion relation for $\tilde{\beta}$:

$$\cos(\tilde{\beta}kd) = \frac{(\beta_2+\beta_1)^2}{4\beta_2\beta_1}\cos\left((\beta_2+\beta_1)k\frac{d}{2}\right) - \frac{(\beta_2-\beta_1)^2}{4\beta_2\beta_1}\cos\left((\beta_2-\beta_1)k\frac{d}{2}\right). \qquad (3)$$

It has two different solutions. One of them is shown graphically in Figure 2 (the parameters of the model are indicated in the insertion) while the other can be obtained by the change of the sign of $\tilde{\beta}$.

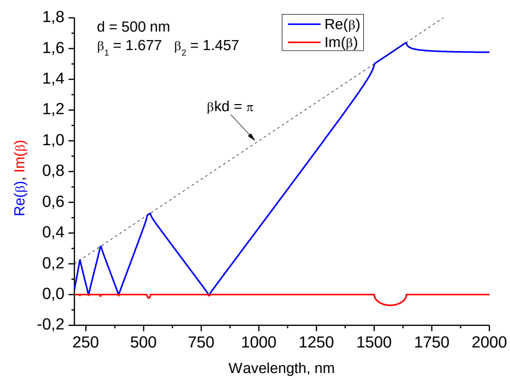

Figure 2. The $\tilde{\beta}$ dependence of the wavelength λ for one of the solutions of dispersion relation (5). The other solution is obtained by changing the sign of $\tilde{\beta}$. The parameters of the model are shown in the insertion.

Bloch functions themselves are determined up to the common complex factor (due to the uniformity of the equations) and read

$$\Phi(y) = \left(e^{i\tilde{\beta}kd} - \frac{\beta_1+\beta_2}{2\beta_1}e^{-i(\beta_1-\beta_2)k\frac{d}{2}} - \frac{\beta_1+\beta_2}{2\beta_1}e^{-i(\beta_1+\beta_2)k\frac{d}{2}}\right)e^{-i(\tilde{\beta}-\beta_1)ky} + \left(e^{i\tilde{\beta}kd} - \frac{\beta_1+\beta_2}{2\beta_1}e^{i(\beta_1+\beta_2)k\frac{d}{2}} - \frac{\beta_1-\beta_2}{2\beta_1}e^{i(\beta_1-\beta_2)k\frac{d}{2}}\right)e^{-i(\tilde{\beta}+\beta_1)ky} \qquad (4)$$

for $y \in [0, d/2]$ and

$$\Phi(y) = -\left(e^{-i\tilde{\beta}kd} - \frac{\beta_2+\beta_1}{2\beta_2}e^{i(\beta_2+\beta_1)k\frac{d}{2}} - \frac{\beta_2-\beta_1}{2\beta_2}e^{i(\beta_2-\beta_1)k\frac{d}{2}}\right)e^{-i(\tilde{\beta}-\beta_2)k(y-d)} - \left(e^{-i\tilde{\beta}kd} - \frac{\beta_2-\beta_1}{2\beta_2}e^{-i(\beta_2-\beta_1)k\frac{d}{2}} - \frac{\beta_2+\beta_1}{2\beta_2}e^{-i(\beta_2+\beta_1)k\frac{d}{2}}\right)e^{-i(\tilde{\beta}+\beta_2)k(y-d)} \quad (5)$$

for $y \in [d/2, d]$. Their domain of definition can be periodically extended to arbitrary number of periods by replacement of y with $(y - Nd)$ (for integer N).

Consider the behavior of obtained solution for various wavelengths λ. For simplicity, suppose there is no absorption (or amplification) in the waveguide. Then the propagation constants β_n are real numbers. The solution of the dispersion relation (3) is different in different spectral regions. Far from the resonances the parameter $\tilde{\beta}$ is real and Bloch functions $\Phi_{\pm}(y)$, corresponding to different sign (+/-) of this parameter, are related with each other by the complex conjugation (the characteristic form of $\Phi_{+}(y)$ is illustrated in Figure 3).

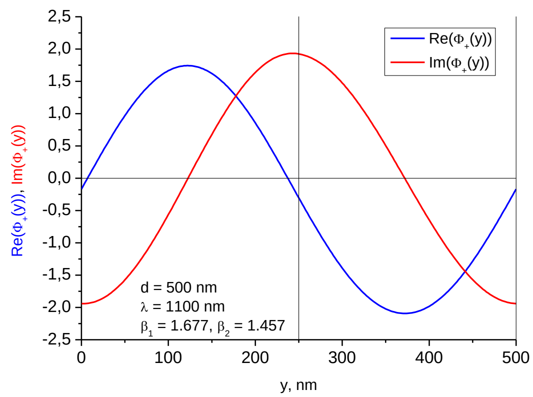

Figure 3. The graph of Bloch function outside the resonance (for λ = 1100 nm). The parameters of the model are shown in the insertion.

The regions of resonances are similar to the forbidden bands in quantum mechanics. In this regions the parameter $\tilde{\beta}$ has non-vanishing imaginary part and the exponential in the equation (2) describes the attenuation of field strength towards the interior of the phase grating. The energy of the incident wave is transmitted to the reflected one (i.e., Bragg resonant reflection occurs). Note, that functions $\Phi_{\pm}(y)$ are not related by complex conjugation, in general. For example, in even resonances, when $Re(\tilde{\beta}) = 0$ these functions are real-valued.

4. Results and Discussion

4.1. Narrow-bandwidth waveguide filters

The constructed model can be employed for describing phase grating interference filters of the finite length L with the reflection maximum in the near IR region of wavelengths, e.g., C-telecom band near 1550 nm.

For modeling such devices one needs to add two uniform waveguides to the left ($y < 0$) and to the right ($y > L$) side of the Bragg grating, presented in Figure 1. In these waveguides complex amplitudes of the electric field strength are given by

$$\vec{E}_l(y,z) = \hat{x}E_i\left(e^{i\beta ky} + R_a e^{-i\beta ky}\right)e(z), \quad \vec{E}_r(y,z) = \hat{x}E_i T_a e^{i\beta ky} e(z) \quad (6)$$

and describe the incident, reflected and transmitted waves. Additional sawing conditions imposed at two resulting interfaces (i.e., at $y = 0$ and $y = L$) allow finding relative contributions of two linear-independent solutions (2) with $\Phi_{+}(y)$ and $\Phi_{-}(y)$ within the grating and determining complex amplitude reflection and transmission coefficients R_a and T_a (the corresponding formulas can be found in Appendix).

The graphs of energy reflection and transmission coefficients $R = |R_a|^2$.and $T = |T_a|^2$ as the functions of the wavelength λ near the main resonance are shown in Figures 4 and 5 . They explicitly demonstrate that for creating interference filters with very narrow transmittance gap ($\Delta\lambda$ ~ 0.1 nm) long Bragg gratings with thousands of periods and low groove height ($\Delta\varepsilon$ ~ 1×10^{-4}) are required (see Figure 4). In contrast, the interference filter with the grating of 50 periods has $\Delta\lambda$ of the order of tens nanometers.

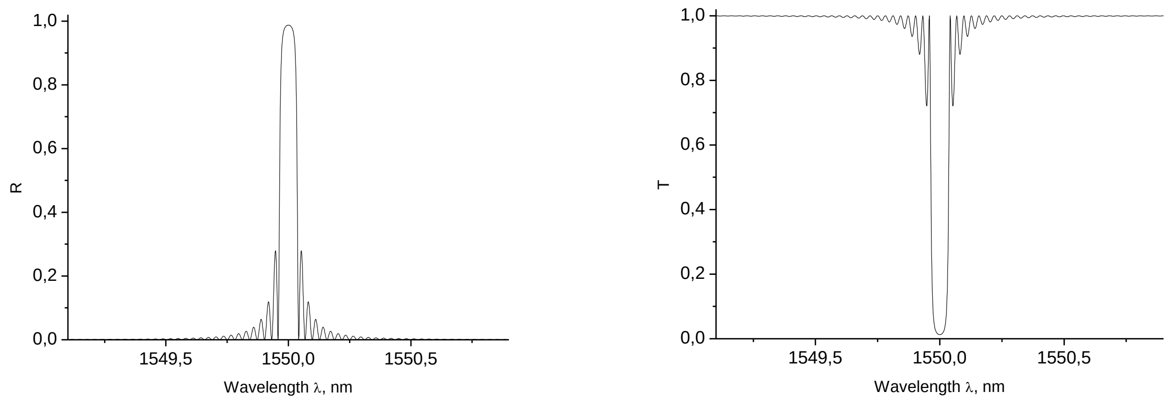

Figure 4. Reflectivity *R* and transmittivity *T* coefficients of the grating with period *d* and length *L* as a function of incident wavelength λ. *d* = 0.58712 мкм, $L = 5 \times 10^4 \times d$, $\Delta\varepsilon = 1 \times 10^{-4}$.

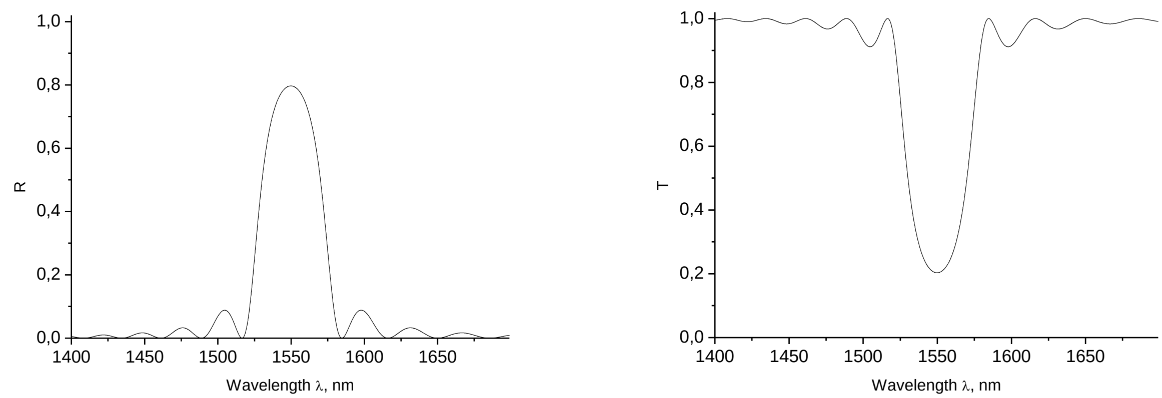

Figure 5. Reflectivity *R* and transmittivity *T* coefficients of the short grating with period *d* and length *L* as a function of incident wavelength λ. *d* = 0.58712 мкм, $L = 50 \times d$, $\Delta\varepsilon = 0.05$.

4.2. Ultrashort light pulses

For studying the interaction of light pulses with Brag gratings within the proposed model, it is convenient to use Gaussian packages. Electric field strength amplitude of the incident radiation is represented by the Fourier integral over monochromatic waves with Gaussian weight

$$\overrightarrow{E_{ip}}(y,t) = Re\int_{\infty}^{\infty}\frac{\tau_0 d\omega}{2\sqrt{\pi}}E_0\hat{x}e^{-\frac{1}{4}(\omega-\omega_0)^2{\tau_0}^2}e^{i\frac{\omega}{c}\beta y-i\omega t}, \quad (7)$$

where τ_0 and ω_0 are the characteristic time and central frequency of the pulse, correspondingly, E_0 denotes its complex amplitude in the center and c is the speed of light in vacuum. For each monochromatic component of the wave package (7) complex amplitude reflactivition and transmission coefficients are known (and given by the equations (8) and (9) in Appendix). Then the reflected (transmitted) field amplitude is determined by the Fourier integral, similar to (6), with additional R_a (T_a) factor in the integrand.

In practice, this integral is taken numerically with partial sum method. The limits of integration are usually chosen far enough from the central frequency to provide the effective descent of the spectral amplitude in the Gaussian package. We used the integration region of $\phi \in [-10,10]$ in terms of dimensionless variable $\phi = \tau_0(\omega - \omega_0)$. The integration step should be small enough to avoid the appearance of spurious Fourier-copies in the time region under investigation (we used $d\phi$ = 0.02).

Average intensity of radiation in the pulse is proportional to the squared absolute value of this integral and is represented in Figure 6 as a function of time. It is calculated for the Gaussian pulse with the duration τ_0= 0.1 ns and the central wavelength of $\lambda_0 = 2\pi c/\omega_0$= 1550.03 nm, incident on the Bragg filter, which has the main reflection resonance at λ = 1550 nm with the characteristic width ~ 0.07 nm (transmission and reflection spectra of this filter are shown in Figure 4).

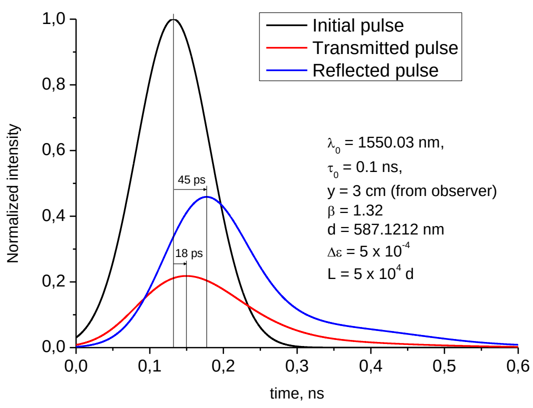

Figure 6. Time dependence of the averaged intensity in the reflected (blue) and transmitted (red) signals for the interference filter exposed to ultrashort Gaussian light pulse (black).

In the considered case of ultrashort pulse the spectral width of Gaussian package (approximately defined by $1/\tau_0$) becomes comparable with the frequency width of the reflection Bragg resonance of the interference filter. The specter of the incident wave changes in the scattering process. This results in the pulse shape distortion and the time delay, observed for the reflected and transmitted signal (see Figure 6).

5. Conclusions

The model of a two-dimensional waveguide interference filter, admitting the exact mathematical solution of Maxwell`s equations, has been considered. It has been employed for calculating the reflectivity and transmittivity coefficients of the particular grating filters and for studying the structure of resonances in them. This model permits to investigate the effects, related with the propagation of light pulses through Bragg gratings, namely the pulse broadening, its time delay and the transformation of the pulse shape.

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Appendix A.

For the completeness we provide the expressions for the complex amplitude reflection and transmission coefficients of the waveguide interference filter with the length L:

$$R_a = -\frac{1}{\Delta}\left(\frac{d\Phi_+}{dy} + ik(\tilde{\beta}-\beta)\Phi_+\right)\Big|_0 \left(\frac{d\Phi_-}{dy} - ik(\tilde{\beta}+\beta)\Phi_-\right)\Big|_L + \frac{1}{\Delta}\left(\frac{d\Phi_+}{dy} + ik(\tilde{\beta}-\beta)\Phi_+\right)\Big|_L \left(\frac{d\Phi_-}{dy} - ik(\tilde{\beta}+\beta)\Phi_-\right)\Big|_0 e^{2i\tilde{\beta}kL} \qquad \text{(A.1)}$$

and

$$T_a = \frac{2ik\beta}{\Delta}\left(\Phi_+\frac{d\Phi_-}{dy} - \Phi_-\frac{d\Phi_+}{dy} - 2ik\tilde{\beta}\Phi_+\Phi_-\right)\Big|_L e^{i(\tilde{\beta}-\beta)kL}. \qquad \text{(A.2)}$$

where the complex parameter Δ is defined as

$$\Delta = \left(\frac{d\Phi_+}{dy} + ik(\tilde{\beta}+\beta)\Phi_+\right)\Big|_0 \left(\frac{d\Phi_-}{dy} - ik(\tilde{\beta}+\beta)\Phi_-\right)\Big|_L - \left(\frac{d\Phi_+}{dy} + ik(\tilde{\beta}-\beta)\Phi_+\right)\Big|_L \left(\frac{d\Phi_-}{dy} - ik(\tilde{\beta}-\beta)\Phi_-\right)\Big|_0 e^{2i\tilde{\beta}kL}. \qquad \text{(A.3)}$$

Bloch functions $\Phi_\pm(y)$ (given by Equations (4) and (5)) and their derivatives should be evaluated either at $y = 0$ or at $y = L$, depending on the subscription.