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Optimization approaches to manage congestions for the phenomenon "Luci D'Artista" in Salerno

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Abstract

This paper focuses on the optimization of traffic flows in case of congestion phenomena due to the event "Luci D'Artista" in Salerno, Italy. The management of traffic deals with two different optimization techniques, that foresee, respectively, a decentralized approach and a genetic algorithm. A cost functional, that estimates the kinetic energy on a portion of the real network of Salerno, is maximized with respect to the distribution coefficients at nodes. The simulation results confirm the decongestion effects, that are also proved via the estimation of the time a car needs to cross fixed paths on the network object of study.

Keywords: Conservation laws; Optimization; Genetic algorithms.

1. Introduction

In modern cities, problems connected to vehicles congestions represent a hard task, that challenges in finding suitable control approaches. Possible solutions consider the adoption of more lanes and the construction of crossings, a situation that is not often possible to manage due to high costs. Issues of heavy traffic provide difficulties inside modern areas of big cities, especially in critical situations of accidents or in occasions of particular events that involve the population, such as the phenomenon "Luci D'Artista" in Salerno, Italy. This last event, during the months of November, December and January, determines an increase in the tourist flow in Salerno thanks to various embellishment operations of the city center via series of lights inside roads and parks.

In this context, using a fluid dynamic model that forecasts the traffic density evolution on road networks (see (Garavello and Piccoli, 2006)), a strategy for the optimal redistribution of traffic at intersections is proposed. Following the adopted model (for the validation, see (Blandin et al., 2009)), the car densities on each road obey a conservation law, while dynamics at road nodes is uniquely solved by using two principal rules: the incoming traffic at nodes distributes to outgoing roads via some distribution coefficients; drivers behave so that the flux through road intersections is maximized.

If an intersection J is of 1×2 type (one incoming road and two outgoing ones), the first rule is easily expressed by a unique distribution parameter α, that indicates the percentage of cars which, from road 1, go to road 2. Assigning initial densities for incoming and outgoing roads and using the second rule, we finally compute the asymptotic solution as function of α.

Here, assuming the distribution coefficient as control parameter, we want to optimize the traffic conditions at intersections of 1×2 type in order to improve urban traffic and face congestions due to the event

"Luci D'Artista" in Salerno. In particular, we analyze the following issue over a fixed time horizon: maximizing a cost functional E, that gives an estimate of the kinetic energy on a portion of the Salerno network.

Different control approaches for the right of way parameters and distribution coefficients have already been treated in (Cascone et al., 2007) and (Cascone et al., 2008), where three cost functionals, dealing with average velocities, average travelling times, and flux, are analyzed for 1×2 and 2×1 intersections. A further analysis is in (Manzo et al., 2012), where coefficients of 2×2 road nodes are optimized for the fast transit of emergency vehicles along assigned paths in case of accidents. For other interesting optimal control problems also based on fluid-dynamics models and its applications to supply chains and arterial systems see (D'Apice et al., 2012), (D'Apice et al., 2013), (Kupenko and Manzo, 2013), (D'Apice et al., 2014), (D'Apice et al., 2016), (Kogut et al., 2016), (Kupenko and Manzo, 2016), (Kupenko and Manzo, 2018), (Manzo, 2019), (Manzo, 2020).

A complete characterization of the cost functionals E on a whole network is very complex, so we follow two different approaches. In the first case, we propose a decentralized approach, i.e. an exact solution is found for single 1×2 nodes and asymptotic E. The global (sub)optimal solution for networks is achieved by localization: the exact optimal solution is applied locally for each time at each junction of 1×2 type. In the second case, we consider a procedure based on a genetic algorithm (GA), whose properties are highly studied in (Berthiau and Siarry, 2001; Kar, 2016; Michalewicz and Janikow, 1991). In particular, the maximization of E involves its numerical computations by variations of traffic coefficients via mechanisms of selection, crossover and mutation.

The optimization results are then proved by simulations (for numerics, see (Godunov, 1959), (Tomasiello, 2011a), (Tomasiello, 2011b), (Tomasiello, 2012)), considering optimal and random distribution coefficients. The first ones are provided by the optimization algorithms, that refer to the two just described approaches. The second ones consider, at the beginning of the simulation process, random values of α, kept constant during the simulation.

The proposed case study is a portion of the Salerno urban network that presents 1×2 and 2×1 road nodes. The simulation results present interesting features: random coefficients frequently cause high congestions, as expected; optimal distribution coefficients allow a redistribution of traffic flows. In particular, random simulation curves of the cost functional E are always lower than the optimal ones due to the decentralized approach and to the GA. The optimal case due to the GA is the highest possible, as the traffic is globally optimized. Finally, using an algorithm for tracing car trajectories on a network, some simulations are run to test how the total travelling time of a driver is influenced by distribution coefficients. As intuition suggests, the time useful to cover a path of a single driver decreases when optimal α values are used.

The benefits of such an analysis are evident: it is possible, for road managers, to understand the future areas in which congestion phenomena are higher in Salerno. Further investigations about car trajectories could be useful to foresee the decision plans to redirect traffic flows inside the urban context.

The paper is organized as follows. Section 2 describes the model for road networks. Section 3 deals with the optimization techniques for the cost functional E. Simulation results are presented in Section 4. Conclusions ends the paper in Section 5.

2. A model for traffic dynamics on road networks

A road network is described as a couple $(\mathcal{A}, \mathcal{B})$, where $\mathcal{A}$ and $\mathcal{B}$ are the set of roads I_k, modeled by intervals $[\mu_k, \theta_k] \subset \mathbb{R}$, $k = 1, \dots, M$, and nodes, respectively.

As for dynamics on roads, assume that, for a generic road I_k, $k = 1, \dots, M$: $\delta_k = \delta_k(t, x) \in \left[0, \delta_{\max}^k\right]$ is the density of vehicles, where $\delta_{\max}^k$ is the maximal density; $f(\delta_k) = \delta_k v(\delta_k)$ is the flux where $v(\delta_k) \in \left[0, v_{\max}^k\right]$ is the average velocity with $v_{\max}^k$ highest possible velocity. Then, the traffic on each road I_k, $k = 1, \dots, M$, is provided by the conservation law (Lighthill-Whitham-Richards model):

$$\frac{\partial \delta_k}{\partial t} + \frac{\partial f(\delta_k)}{\partial x} = 0. \tag{1}$$

For each road I_k, $k = 1, \dots, M$, we get the following: (H) f is a strictly concave C^2 function such that $f(0) = f\left(\delta_{\max}^k\right) = 0$. Fixing the decreasing velocity function:

$$v(\delta_k) = v_{\max}^k \left(1 - \frac{\delta_k}{\delta_{\max}^k}\right), \delta_k \in \left[0, \delta_{\max}^k\right], \tag{2}$$

a flux function, that respects (H), is, for $v_{\max}^k = \delta_{\max}^k = 1$:

$$f(\delta_k) = \delta_k (1 - \delta_k), \quad \delta_k \in [0, 1], \tag{3}$$

that has a unique maximum $\sigma = \frac{1}{2}$.

In order to define the dynamics at junctions, we consider Riemann Problems (RPs), namely Cauchy Problems with a constant initial datum for each incoming and outgoing road. Fix a node J of $n \times m$ type (n incoming roads I_r, $r = 1, \dots, n$, and m outgoing roads, I_s, $s = n+1, \dots, n+m$) and an initial datum $\delta_0 = (\delta_{1,0}, \dots, \delta_{n,0}, \delta_{n+1,0}, \dots, \delta_{n+m,0})$.

A Riemann Solver (RS) for J is a map $RS : [0,1]^n \times [0,1]^m \to [0,1]^n \times [0,1]^m$ that associates to δ_0 a vector $\widehat{\delta} = \left(\widehat{\delta}_1, \ldots, \widehat{\delta}_{n,0}, \widehat{\delta}_{n+1}, \ldots, \widehat{\delta}_{n+m}\right)$ so that the the wave $\widetilde{\delta}_r = \left(\delta_{r,0}, \widehat{\delta}_r\right)$ is solution for the incoming road I_r, $r = 1, \ldots, n$, and the wave $\widetilde{\delta}_s = \left(\delta_{s,0}, \widehat{\delta}_s\right)$ is solution for the outgoing road I_s, $s = n+1, \ldots, n+m$. The following conditions hold: (H1) $RS(RS(\delta_0)) = RS(\delta_0)$; (H2) for a generic outgoing (resp. incoming) road, the wave $\widetilde{\delta}_s$ (resp. $\widetilde{\delta}_r$) has positive (resp. negative) speed.

If $m \geq n$, a possible RS at J is described through the rules:

(A) Traffic distributes at J according to parameters, that are collected in a traffic matrix $A = (\alpha_{s,r})$, $r = 1, \ldots, n$, $s = n+1, \ldots, n+m$, $0 < \alpha_{s,r} < 1$, $\sum_{s=n+1}^{n+m} \alpha_{s,r} = 1$. The r–th column of A provides the percentages of traffic that, from the incoming road I_r, goes to the outgoing roads;

(B) Fullfilling (A), drivers maximize the flux through J.

If $n > m$, a further rule (yielding criterion) is necessary:

(C) If R is the amount of cars that can enter the outgoing roads, then $p_r R$ cars come from the incoming road I_r, where $p_r \in \,]0,1[$, $\sum_{r=1}^{n} p_r = 1$, indicates the right of way parameter for road I_r, $r = 1, \ldots, n$.

An example of possible Riemann Solver is described as follows (for other cases, see (Garavello and Piccoli, 2006)).

For nodes J of 1×2 type (incoming road I_1, and outgoing roads I_2 and I_3), indicate the densities of cars for incoming and outgoing roads, respectively, by $\delta_1(t,x) \in [0,1]$, $(t,x) \in \mathbb{R}^+ \times I_1$, and $\delta_s(t,x) \in [0,1]$, $(t,x) \in \mathbb{R}^+ \times I_s$, $s = 2,3$. From (H2), for the flux function (3) and initial datum of an RP at J represented by $\delta_0 = (\delta_{1,0}, \delta_{2,0}, \delta_{3,0})$, we have that the maximal flux values on roads are:

$$\gamma_u^{\max} = \begin{cases} f(\delta_{u,0}), & \text{if } 0 \leq \delta_{u,0} \leq \frac{1}{2} \text{ and } u = 1, \\ & \text{or } \frac{1}{2} \leq \delta_{u,0} \leq 1 \text{ and } u = 2,3, \\ f\left(\frac{1}{2}\right), & \text{if } \frac{1}{2} \leq \delta_{u,0} \leq 1 \text{ and } u = 1, \\ & \text{or } 0 \leq \delta_{u,0} \leq \frac{1}{2} \text{ and } u = 2,3. \end{cases}$$

In this case, matrix A has the only coefficients $\alpha_{2,1} = \alpha$, $\alpha_{3,1} = 1 - \alpha_{2,1} = 1 - \alpha$. From rules (A) and (B), we get that the flux solution to the RP at J is $\widehat{\gamma} = (\widehat{\gamma}_1, \widehat{\gamma}_2, \widehat{\gamma}_3)$, where:

$$\widehat{\gamma}_1 = \min\left\{\gamma_1^{\max}, \frac{\gamma_2^{\max}}{\alpha}, \frac{\gamma_3^{\max}}{1-\alpha}\right\},$$

$$\widehat{\gamma}_2 = \alpha\widehat{\gamma}_1, \quad \widehat{\gamma}_3 = (1-\alpha)\widehat{\gamma}_1.$$

Once $\widehat{\gamma}$ is known, $\widehat{\delta}$ is found as:

$$\widehat{\delta}_u \in \begin{cases} \{\delta_{u,0}\} \cup \left]\tau(\delta_{u,0}), 1\right], & \text{if } 0 \leq \delta_{u,0} \leq \frac{1}{2} \text{ and } u = 1, \\ & \text{or } \frac{1}{2} \leq \delta_{u,0} \leq 1 \text{ and } u = 2,3, \\ \left[0, \frac{1}{2}\right], & \text{if } 0 \leq \delta_{u,0} \leq \frac{1}{2}, u = 2,3, \\ \left[\frac{1}{2}, 1\right], & \text{if } \frac{1}{2} \leq \delta_{u,0} \leq 1, u = 1, \end{cases}$$

where $\tau : [0,1] \to [0,1]$ is the map such that $f(\tau(\delta)) = f(\delta)\ \forall\, \delta \in [0,1]$ and $\tau(\delta) \neq \delta\ \forall\, \delta \in [0,1] \setminus \{\frac{1}{2}\}$.

Further details about the adopted model are in (Garavello and Piccoli, 2006).

3. Optimization

Fix a road network $(\mathcal{A}, \mathcal{B})$ as described in previous Section. The network performances are optimized by the following cost functional:

$$E(t) := \sum_{k=1}^{M} \int_{I_k} f(\delta_k(t,x))\, v(\delta_k(t,x))\, dx, \qquad (4)$$

that is a term proportional to the kinetic energy on the whole network.

Assuming bounded $\delta_k(t,x)$, $k = 1, \ldots, M$, the aim is to maximize $E(t)$ with respect to traffic distribution parameters at each node $J \in \mathcal{B}$. In what follows, we propose two different approaches.

3.1. First approach

As the solution of the optimization control problem considers space-time variables, we refer to a heuristic defined by the following steps:

Step 1 Fix a node $J \in \mathcal{B}$ of $n \times m$ type, the initial datum $\left(\delta_{1,0}^J, \ldots, \delta_{n,0}^J, \delta_{n+1,0}^J, \ldots, \delta_{n+m,0}^J\right)$ at J, and define the local cost functional:

$$E_J(t) := \sum_{k=1}^{n+m} \int_{I_k} f\left(\delta_k^J(t,x)\right) v\left(\delta_k^J(t,x)\right) dx.$$

Step 2 For a fixed time horizon $[0,T]$, with T sufficiently big, assume traffic distribution coefficients and right of way parameters as controls, and maximize $E_J(t)$ with respect to them.

Step 3 Construct the optimal solution of the overall network by localization, i.e. by using the single optimization solutions at each node $J \in \mathcal{B}$ of $n \times m$ type.

Assume that $J \in \mathcal{B}$ is of 1×2 type. In this case, the traffic distribution matrix is defined by a unique parameter α (see the previous section) and, for step 2, we get the following conditions:

- $\mathbf{C}_1 : \gamma_3^{\max} \leq \frac{\gamma_1^{\max}}{2} < \gamma_1^{\max} \leq \gamma_2^{\max}$;

- $\mathbf{C}_2 : \gamma_2^{\max} < \frac{\gamma_1^{\max}}{2} < \gamma_1^{\max} \leq \gamma_3^{\max}$;
- $\mathbf{C}_3 : \gamma_2^{\max} < \gamma_3^{\max} < \gamma_1^{\max}$;
- $\mathbf{C}_{3A} : \gamma_1^{\max} - \gamma_3^{\max} \geq \gamma_2^{\max}$;
- $\mathbf{C}_{3B} : \gamma_1^{\max} - \gamma_3^{\max} < \gamma_2^{\max} \leq \frac{\gamma_1^{\max}}{2}$;
- $\mathbf{C}_4 : \gamma_3^{\max} < \gamma_2^{\max} < \gamma_1^{\max}$;
- $\mathbf{C}_5 : \frac{\gamma_1^{\max}}{2} \leq \gamma_1^{\max} - \gamma_3^{\max} < \gamma_2^{\max}$,

and define:

$$r_{ij}^{\max} := \frac{\gamma_i^{\max}}{\gamma_j^{\max}}.$$

We have the following:

Theorem 1 *Fix a node $J \in \mathcal{B}$ is of 1×2 type. Assuming T sufficiently big, the cost functional $E_J(t)$ is maximized for the following value α_J^{opt} (in some case, the optimal control does not exist but it is approximated using the small and positive constant ε):*

$$\alpha_J^{opt} = \begin{cases} 1 - r_{31}^{\max} + \varepsilon, & \text{if } \mathbf{C}_1 \text{ is satisfied;} \\ r_{21}^{\max}, & \text{if } \mathbf{C}_2 \text{ holds;} \\ \frac{1}{1+r_{32}^{\max}}, & \text{if } \mathbf{C}_3 \text{ and } \mathbf{C}_{3A} \text{ both hold;} \\ r_{21}^{\max} - \varepsilon, & \text{if } \mathbf{C}_3 \text{ and } \mathbf{C}_{3B} \text{ are both true;} \\ \frac{1}{1+r_{32}^{\max}} + \varepsilon, & \text{if } \mathbf{C}_{3A} \text{ and } \mathbf{C}_4 \text{ both hold;} \\ 1 - r_{31}^{\max} - \varepsilon, & \text{if } \mathbf{C}_5 \text{ holds;} \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

3.2. Second approach

A possible optimization of (4) can be obtained via a Genetic Algorithm (GA). Convergence issues of such algorithms are properly considered in (Barrios et al., 1998; Cerf, 1998).

For a maximal number of iterations Ψ, the algorithm works as follows.

At the iteration 0, generate an initial population indicated by $C^0 = \left(C_1^0, C_2^0, \ldots, C_P^0\right)$, seen as a set of possible controls (distribution coefficients and/or right of way parameters), and compute the value $\Omega_0 := E\left(C_1^0, C_2^0, \ldots, C_P^0\right)$ of the fitness function (4).

In general, indicating by $\Omega_k := E\left(C_1^k, C_2^k, \ldots, C_P^k\right)$ the value of (4) at the iteration k, $k \geq 1$, the steps of such iteration are:

Step 1 starting from $C^{k-1} = \left(C_1^{k-1}, C_2^{k-1}, \ldots, C_P^{k-1}\right)$, get $\overline{C^k} = \left(\overline{C_1^k}, \overline{C_2^k}, \ldots, \overline{C_P^k}\right)$ via mechanisms of selection, crossover and mutation;

Step 2 compute the value $\overline{\Omega_k} := E\left(\overline{C_1^k}, \overline{C_2^k}, \ldots, \overline{C_P^k}\right)$ of (4);

Step 3 if $\overline{\Omega_k} < \Omega_{k-1}$, set $C^k := \overline{C^k}$ and go to step 4; otherwise, go back to step 1;

Step 4 set $k := k + 1$ and go back to step 1 if $k \leq \Psi$; otherwise, stop.

4. Simulations

In this section the simulation of a part of the urban network of Salerno, Italy, is presented. The network topology is represented in Figure 1, and has four principal roads, that are divided into segments labelled by letters. Precisely, we have Via Torrione (segments a, b, c), Via L. Vinciprova (segments d, e), Via S. Mobilio (segments f, g, h, i), and Via L. Guercio (segment l).

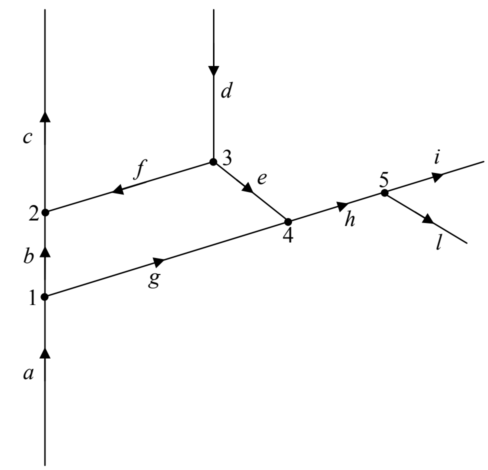

Figure 1. Topology of the portion of the real urban network of Salerno.

We have inner roads segments (b, e, f, g, h), and external ones (a, c, d, i, l). Road nodes are indicated by numbers and, precisely, 1, 3, and 5 are of 1×2 type, while 2 and 4 are of 2×1 type. Traffic flows are simulated by the Godunov method with $\Delta x = 0.0125$, $\Delta t = 0.5\Delta x$ in a time interval $[0, T]$, with $T = 150$ min. Initial conditions and boundary data for densities are in Table 1, and have been estimated by historical series during the month of December 2019.

Table 1. Initial conditions (IC) and boundary data (BD, normalized in $[0, 1]$); right of way parameters (RWP).

Road	IC	BD	RWP
a	0.15	0.35	/
b	0.15	/	0.15
c	0.15	0.95	0.25
d	0.15	0.35	/
e	0.85	/	/
f	0.15	/	0.75
g	0.8	/	0.65
h	0.65	/	/
i	0.8	0.95	/
l	0.8	0.95	/

Notice that, for road intersections 2 and 4, right of way parameters are chosen according to measures on the real network.

Precisely, the features of the provided simulations are as follows:

- *Data collection*: historical series provided by the municipality of Salerno.
- *Case study definition (network to simulate)*: a part of Salerno network, see Figure 1.
- *Implementation*: C++ code, where data structures are useful to store the network graph; Godunov method for the approximation of conservation laws; ad hoc functions, realized by the author, for the resolution of linear programming problems at the nodes of the network.
- *Parameters for simulations*: see Table 1.
- *Validation*: not yet realized within the scenario of Salerno.

Notice that validation steps have not been done due to the region Campania's lockdown for COVID-19 emergency. Future research activities aim at this purpose, though some validation processes are described in (Blandin et al., 2009) for Rome (Italy).

4.1. Discussion and results

In this subsection, we discuss the obtained results.

In Figure 2, we report the behaviour of E, where optimal simulations are indicated again by continuous lines, while random cases by dashed ones. Random simulations curves of E are always lower than the optimal ones. In fact, when optimal parameters are used, a flows redistribution occurs on roads, with consequent reduction of congestions at junctions of 1×2 type. Notice that the optimal behaviour with GA, labelled by GA, is higher, as expected, by the optimal curve obtained through the decentralized approach, labelled by DA.

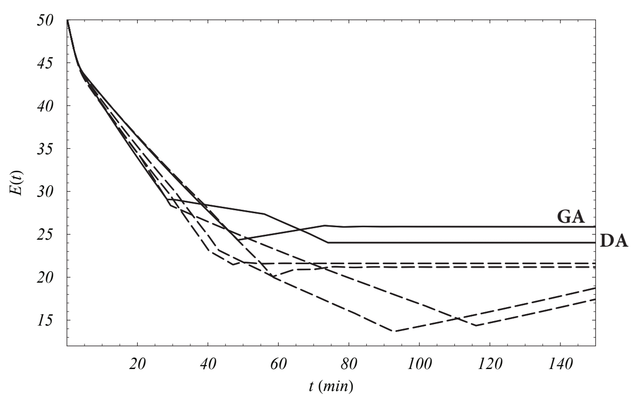

Figure 2. Evolution in $[0, T]$ of $E(t)$, evaluated for optimal distribution coefficients (continuous line) and random choices (dashed lines).

Assume that a car travels along a path in a network, whose traffic dynamics is provided by (1). The position of the driver $x = x(\tau)$ is obtained by the Cauchy problem:

$$\begin{cases} \dot{x} = v(\delta(\tau, x)), \\ x(\tau_0) = x_0, \end{cases} \tag{5}$$

where x_0 is the initial position at the initial time τ_0. Via suitable numerical methods, described in (Tomasiello, 2011a), (Tomasiello, 2011b) and (Tomasiello, 2012), we aim to estimate the driver travelling time and to prove the goodness of the optimization results. Precisely, we compute the trajectory along road g and the time to cover it in optimal and random conditions.

In Figure 3, we assume that the car starts its own travel at the beginning of road g at the initial time $\tau_0 = 55$ and compute the trajectories $x(\tau)$ in the optimal case with GA (continuous line) and in random cases (dashed lines).

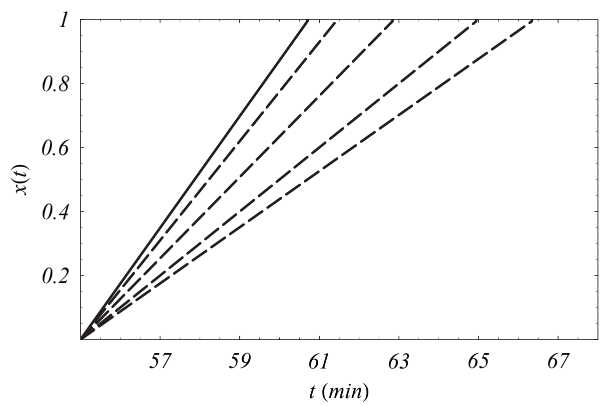

Figure 3. Trajectory $x(\tau)$ along road g with $t_0 = 55$; optimal coefficients (continuous line) and random choices (dashed lines).

The behaviour $x(\tau)$ in the optimal case has always a higher slope than the trajectories in random cases as traffic levels are low. When random parameters are considered, shocks propagating backwards increase the density values on the network. The velocity for cars is reduced and exit times from road g become longer. Assuming $\tau_0 = 55$ we have the following time instants τ_{out} in which the car goes out of road g, either for the optimal distribution coefficients with GA (τ_{out}^{opt}) or random choices ($\tau_{out}^{r_i}$, r_i, $i = 1, .., 4$): $\tau_{out}^{opt} = 60.70$, $\tau_{out}^{r_1} = 66.34$, $\tau_{out}^{r_2} = 64.94$, $\tau_{out}^{r_3} = 62.84$ and $\tau_{out}^{r_4} = 61.42$.

5. Conclusions

In this paper, an optimization study has been described in order to manage car traffic in case of the event "Luci D'Artista" in Salerno, Italy. In particular, suitable distribution coefficients at road intersections have been found via a cost functional, that deals with the kinetic energy for a portion of the real urban network of Salerno. The obtained optimization results have been useful to prove that, in some cases, a total decongestion effect occurs. This is also confirmed by simulations of cars trajectories on some roads: optimal distribution parameters allow to reduce the times needed to cover fixed paths on the network object of study.

Future research activities aim at the following steps: a validation of the model within the context of traffic inside Salerno, an activity that was suspended because of the National Health Emergency due to COVID-19; definition of new cost functionals for the analysis of performances of traffic; new swarm intelligence optimization algorithms, based on natural dynamics of ants and bees.

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