



Capacitated Vehicle Routing Problem with Time Windows: a Linear Model and a Case Study of Express Courier

Giorgia Casella¹, Sara Carattini¹, Teresa Murino², Letizia Tebaldi¹ and Eleonora Bottani^{1,*}

¹Department of Engineering and Architecture, University of Parma, Parco Area delle Scienze, 181/A, 43124, Parma (Italy).

²Department of Chemical, Materials and Industrial Production Engineering, University of Naples “Federico II”, Piazzale V. Tecchio, 80, 80125, Napoli (Italy).

*Corresponding author. Email address: eleonora.bottani@unipr.it

Abstract

Given the importance gained by the e-commerce field in the recent years, this study investigates the issue of minimizing the delivery travel time of a real company located in the South of Italy and operating as a courier, express and parcel (CEP) service provider. The scenario under examination consists of a depot, three vehicles and several customers served by the CEP company. A Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) model is formulated to optimize the deliveries to the customers for the targeted company and solved under the commercial software IBM ILOG CPLEX Optimization Studio. As outcomes, the model returns a simulated path covered by the vehicles and computes the corresponding travel time. Results show that with the proposed formulation, the time windows (TWs) of all customers are respected. Because the analysis is grounded on a real company, the results are expected to provide practical indications to logistics and supply chain managers, to maximize the performance of their delivery system.

Keywords: Capacitated vehicle routing problem; time windows; simulation; express courier.

1. Introduction

With the rapid development of social progress, economic globalization and information networks, market competition is today very intense, and e-commerce is increasingly affecting every day's economic life (Fu et al. 2019; Yang and Li 2013). Therefore, the express logistics industry has been an increasingly more crucial part in our lives (Yang et al. 2013) with postal and courier services that are assigned to carry documents, parcels or other items

from one place to another quickly and efficiently (Fu et al. 2019; Izzah, Rifai, and Yao 2016). Courier routing and scheduling is an important concern for organizations that are highly dependent on their logistics system (Zangeneh-Khmooshi, Zabinsky and Heim 2013). There is strong pressure for improving the efficiency of logistics services as a result of rising energy costs and strong competition among carriers (Purnamasari and Santoso 2018). This cost pressure forces the courier, express and parcel (CEP) service providers to optimize their logistics networks continuously (Kunkel and Schwind 2011). In particular,



courier delivery services deal with the problem of routing a fleet of vehicles from a depot to serve a set of customers that are geographically dispersed (Wang, Ordóñez and Dessouky 2012). In addition, the logistics service is faced with new challenges. Firstly, the scale of customers has been enlarged, resulting in more difficult NP-hard problems. Secondly, the customers have requirements of time windows (TWs), including both the basic TWs and the expected ones (Yang et al. 2013). In other words, the delivery problem for couriers is an extension of vehicle routing problem (VRP). Indeed, VRP is a generic name for a class of problems in which a set of routes for a fleet of vehicles based on one or several depots are to be determined for a number of geographically dispersed points (customers, stores, schools, cities, warehouses, etc.) at the minimum cost (distance, time, or any other desired factor) (Sun et al. 2018; Khouadjia et al. 2012). Dantzing and Ramser (1959) were the first authors who introduced the “Truck Dispatching Problem”, which can be considered as a generalization of the “Traveling-Salesman Problem”. Five years later, Clarke and Wright (1964) generalized this problem to a linear optimization problem that is commonly encountered in the domain of logistics and transport (Armenzoni et al. 2017). This problem was subsequently named the VRP and is currently one of the most widely studied topics in the field of operations research (Braekers, Ramaekers and Nieuwenhuys 2016). Now, VRP offers a wealth of heuristic and metaheuristic approaches, which are surveyed in various papers (Laporte 1992; Gendreau, Laporte and Potvin 2002; Cordeau et al. 2005; Panneerselvam and Kumar 2015). Some variants of VRP often studied are the Capacitated Vehicle Routing Problem (CVRP), the Vehicle Routing Problem with Time Windows (VRPTW) and the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) (Rochman, Prasetyo and Nugroho 2017). In the CVRP each vehicle has a maximum capacity and the customers have a demand which must be satisfied (Gonzalez et al. 2017), whereas in the VRPTW each client must be served within a predefined TW, whose boundaries are the earliest and the latest time when the goods can be delivered to or collected from the customer (Korcyl, Książek and Gdowska 2016). Finally, the CVRPTW is the CVRP, in which, additionally, each customer has a TW for being served (Gonzalez et al. 2017).

Despite the relevance of these concepts, however, few of the research relating to VRP has been focused on CEP services (López-Santana, Rodríguez-Vásquez and Méndez-Giraldo 2018). In line with this consideration, the aim of this paper is to contribute to the literature by evaluating and minimizing the total travel time of an existing company in the south of Italy, which operates as a CEP service provider. A CVRPTW model is developed to this end using IBM ILOG CPLEX Optimization Studio, and used to support the simulated analysis, i.e. to minimize the total travel time for each vehicle and simultaneously satisfy the

TWs of customers.

The remainder of the paper is organised as follows. The next section describes the CVRPTW model in detail. Section 3 provides some information about the case study under examination. Section 4 shows the logic used to carry out the computational study. In section 5, we report the main results. Section 6 presents the conclusions and the directions for future research activities.

2. Definition and formulation of the CVRPTW

The CVRPTW proposed in this study is as follows. A fleet of vehicles available at the distribution centre is to be assigned to deliver goods to a given number of customers. The locations of the distribution centre and the customers are all fixed and known. The distance between the nodes (including the distribution centre and all customers) is determined by their locations. Each vehicle has a fixed load capacity and a maximum allowable working time each day. Each customer has a fixed service TW and must be assigned to exactly one distribution trip which originates and ends at the distribution centre; on the contrary, each distribution trip can obviously visit more customers in sequence. Each distribution trip must be covered by exactly one vehicle. The travel times between the nodes depend on the distances between them. The solution of the problem involves determining the sequence of customers to be visited in each distribution trip and the vehicles' working timetable.

The objective is to minimize the total scheduling time (including the travel time between the nodes and the waiting/servicing time at the customer's site) of all vehicles considering the fixed TW.

The constraints of the CVRPTW model are as follows:

- 1- Capacity constraints: the total customers' demand in one distribution trip should not exceed the load capacity of the service vehicle;
- 2- TW constraints: the vehicle should arrive earlier than the upper bound and later than the lower bound of the service TW.

The nomenclature used in the analysis is shown in Table 1.

To minimize the total scheduling time of all vehicles considering the fixed service time, a CVRPTW model was constructed with capacity constraints and TWs. A weighted complete graph $G = (V, C)$ is given where $C = \{i_0, i_1, i_2, \dots, i_n, i_{n+1}\}$ is the set of n customers in which i_0 and i_{n+1} represent the depot of origin and destination, respectively, while V is the fleet of vehicles. The set of all vertices, that is, $0, 1, \dots, n + 1$ is denoted as N . Consider the set of arcs A that represent the connections between the depot and the customers, but also between the customers themselves.

Table 1. Nomenclature.

| Symbol | Description | Unit Of Measurement |
|-------------------|--|---------------------|
| Subscripts | | |
| i, j | Customers ($i, j = 0, 1, \dots, n, n + 1$) | - |
| Parameters | | |
| C | Set of n customers | - |
| V | Fleet of vehicle | - |
| A | Set of arcs | - |
| N | Set of vertices | - |
| c_{ij} | A non-negative cost | - |
| t_{ij} | Travel time | Min |
| q | Vehicle capacity | Kg |
| d_i | Demand of customer i | Kg |
| a_i, b_i | Lower and upper limit of the time window | Time format |
| x_{ijk}, s_{ik} | Decision variables | - |
| v | Vehicle | - |
| t_s | Service time | Min |

No arcs end at vertex 0 and none originates at vertex $n + 1$. A non-negative cost c_{ij} is associated with each arc $(i, j) \in A, i \neq j$, reflecting the time (t_{ij}) required to cover the same arc. Each vehicle is characterized by a capacity q and each customer i by a demand d_i . In addition, a TW $[a_i, b_i]$ is known for each customer. The vehicle must arrive at the customer's site before the upper limit b_i of the TW; it may arrive even before the time a_i , but the customer cannot be served, so that the vehicle must wait. The depot is also characterized by a TW indicated with $[a_0, b_0]$ which denotes the starting/ending time of the depot's activities. The model includes two sets of decision variables x_{ijv} and s_{iv} . For each arc $(i, j) \in A, i \neq j, j \neq n + 1, j \neq 0$, and each vehicle v we define x_{ijv} as

$$x_{ijv} = \begin{cases} 1, & \text{if vehicle } v \text{ drives from vertex } i \text{ to vertex } j \\ 0, & \text{otherwise} \end{cases}$$

The decision variable s_{iv} is defined for each vertex i and each vehicle v and denotes the time when the vehicle v starts to serve customer i . In case vehicle v does not serve customer i , s_{iv} has no meaning and consequently its value is irrelevant. We assume $a_0 = 0$ and therefore $s_{0v} = 0$, for all v .

The goal is to design a set of routes that minimizes the total cost, so that each customer is served exactly once, every route originates at vertex 0 and ends at vertex $n + 1$, and the TW of the customers and capacity constraints of the vehicles are met.

The CVRPTW can be formulated mathematically as follows:

$$\text{Minimize } z = \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijv} \quad (1)$$

subject to:

$$\sum_{v \in V} \sum_{i \in N} \sum_{j \in N} x_{ijv} = 1, \forall i \in C \quad (2)$$

$$\sum_{i \in C} d_i \sum_{j \in N} x_{ijv} \leq q, \forall v \in V \quad (3)$$

$$\sum_{j \in N} x_{0jv} = 1, \forall v \in V \quad (4)$$

$$\sum_{i \in N} x_{ihv} - \sum_{j \in N} x_{hjv} = 0, \forall h \in C, \forall v \in V \quad (5)$$

$$\sum_{i \in N} x_{i, n+1, v} = 1, \forall v \in V \quad (6)$$

$$s_{iv} + t_{ij} - v(1 - x_{ijv}) \leq s_{jv}, \forall i, j \in N, \forall v \in V \quad (7)$$

$$a_i \leq s_{jv} \leq b_i, \forall i \in N, \forall v \in V \quad (8)$$

$$x_{ij} \in \{0, 1\}, \forall i, j \in N, \forall v \in V \quad (9)$$

The objective function (1) minimizes the total travel cost. Constraint (2) ensures that each customer is visited exactly once, while constraint (3) states that a vehicle can only be loaded up to its capacity. Next, equations (4), (5) and (6) indicate that each vehicle must leave the depot 0; once a vehicle has arrived at a customer, it has to leave for another destination; and finally, all vehicles must arrive at the depot (node $n + 1$). The inequality (7) establishes the relationship between the vehicle departure time from a customer and its immediate successor. Finally, constraint (8) ensures that the TWs are met, and constraint (9) defines the decision variable used (Kallehaug et al. 2005).

3. The case study

The case study company examined in this paper will be referred to as "Company A" (for the sake of confidentiality) and provides ordinary and express shipping services in Mariglianella, by managing transport and delivery of products of various sizes in the Neapolitan territory (Southern Italy). This company is taken as a case study for investigating the CVRPTW for a CEP, with the aim to minimize the total time of goods delivery to the established customers, at the same time meeting the constraints of the fixed TWs. The company serves 29 main customers in the areas of Sorrento, Gragnano, Agerola, Pompei and Castellammare di Stabia (Figure 1).



Figure 1. Areas under study.

Table 2 shows the list of deliveries expected by Company A for 28 February 2019, a sample day

considered in the case study for which the routes were simulated. The depot is indicated as node 1, while each customer is denoted by a number from 2 to 30 that identifies the corresponding node. Further data available for each customer are: the product demand [kg], the TW [in time format and respective minutes] and the number of parcels. In the light of the TWs of each customer, the opening time of the depot has been set at 9.00 am.

Table 2. List of deliveries.

| Node | d_i [Kg] | Tw [a_i, b_i] | Number Of Parcels |
|------|------------|---------------------------------------|-------------------|
| 1 | - | [9:00;18:00] \approx [0;540] min | - |
| 2 | 14.60 | [10:00;18:00] \approx [60;540] min | 1 |
| 3 | 10.00 | [10:00;18:00] \approx [60;540] min | 1 |
| 4 | 27.40 | [10:00;18:00] \approx [60;540] min | 3 |
| 5 | 2.20 | [10:00;18:00] \approx [60;540] min | 1 |
| 6 | 4.30 | [10:00;18:00] \approx [60;540] min | 1 |
| 7 | 69.20 | [10:00;18:00] \approx [60;540] min | 2 |
| 8 | 45.52 | [10:00;18:00] \approx [60;540] min | 2 |
| 9 | 0.30 | [10:00;18:00] \approx [60;540] min | 1 |
| 10 | 18.80 | [10:00;18:00] \approx [60;540] min | 1 |
| 11 | 5.40 | [10:00;18:00] \approx [60;540] min | 2 |
| 12 | 29.70 | [10:00;18:00] \approx [60;540] min | 2 |
| 13 | 52.00 | [10:00;12:00] \approx [60;180] min | 2 |
| 14 | 7.40 | [10:00;18:00] \approx [60;540] min | 1 |
| 15 | 10.50 | [10:00;18:00] \approx [60;540] min | 1 |
| 16 | 5.10 | [10:00;18:00] \approx [60;540] min | 1 |
| 17 | 3.50 | [10:00;18:00] \approx [60;540] min | 1 |
| 18 | 5.90 | [10:00;18:00] \approx [60;540] min | 1 |
| 19 | 14.00 | [10:00;18:00] \approx [60;540] min | 1 |
| 20 | 1.00 | [10:00;18:00] \approx [60;540] min | 2 |
| 21 | 2.10 | [10:00;18:00] \approx [60;540] min | 1 |
| 22 | 0.90 | [10:00;18:00] \approx [60;540] min | 1 |
| 23 | 26.00 | [10:00;18:00] \approx [60;540] min | 2 |
| 24 | 3.90 | [10:00;18:00] \approx [60;540] min | 2 |
| 25 | 32.00 | [10:00;18:00] \approx [60;540] min | 1 |
| 26 | 0.20 | [10:00;18:00] \approx [60;540] min | 1 |
| 27 | 1.00 | [10:00;18:00] \approx [60;540] min | 1 |
| 28 | 6.80 | [12:00;13:00] \approx [180;240] min | 1 |
| 29 | 1.30 | [10:00;14:00] \approx [60;300] min | 1 |
| 30 | 3.00 | [10:00;18:00] \approx [60;540] min | 1 |

Table 3 shows the matrix of the travel times between the depot and the customers and between the customers themselves. The travel times have been evaluated according to the traffic level of the different daily bands and do not include the waiting/servicing time at each customer's site.

Table 3. Matrix of the travel times.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | | |
| 1 | 0 | 13 | 34 | 38 | 36 | 36 | 32 | 35 | 34 | 37 | 36 | 35 | 35 | 36 | 34 | 34 | 36 | 37 | 35 | 36 | 35 | 34 | 36 | 36 | 38 | 36 | 35 | 35 | 35 | | |
| 2 | 33 | 0 | 7 | 3 | 7 | 7 | 7 | 12 | 5 | 7 | 4 | 5 | 8 | 8 | 7 | 11 | 11 | 9 | 18 | 8 | 8 | 9 | 11 | 7 | 7 | 10 | 3 | 2 | 2 | 4 | |
| 3 | 34 | 7 | 0 | 5 | 6 | 6 | 13 | 4 | 6 | 3 | 4 | 7 | 7 | 6 | 10 | 10 | 8 | 13 | 7 | 7 | 8 | 10 | 6 | 6 | 9 | 2 | 1 | 1 | 5 | | |
| 4 | 38 | 3 | 12 | 0 | 4 | 4 | 5 | 12 | 2 | 4 | 1 | 2 | 5 | 5 | 4 | 8 | 8 | 6 | 10 | 5 | 5 | 6 | 8 | 4 | 4 | 7 | 3 | 4 | 2 | 8 | |
| 5 | 36 | 6 | 6 | 4 | 0 | 1 | 5 | 11 | 1 | 4 | 3 | 2 | 4 | 4 | 4 | 7 | 5 | 9 | 4 | 4 | 5 | 7 | 3 | 4 | 7 | 6 | 7 | 6 | 10 | | |
| 6 | 36 | 8 | 6 | 4 | 1 | 4 | 11 | 2 | 4 | 3 | 2 | 4 | 4 | 4 | 4 | 7 | 7 | 5 | 9 | 4 | 4 | 5 | 7 | 3 | 4 | 7 | 6 | 7 | 5 | 10 | |
| 7 | 36 | 6 | 6 | 4 | 4 | 4 | 10 | 3 | 4 | 4 | 4 | 2 | 2 | 1 | 4 | 4 | 2 | 6 | 1 | 2 | 2 | 4 | 1 | 1 | 6 | 8 | 8 | 7 | 9 | | |
| 8 | 32 | 13 | 12 | 11 | 10 | 11 | 10 | 10 | 9 | 11 | 11 | 10 | 9 | 11 | 7 | 8 | 11 | 11 | 10 | 11 | 10 | 11 | 10 | 11 | 10 | 13 | 15 | 15 | 14 | 8 | |
| 9 | 35 | 5 | 4 | 2 | 1 | 1 | 3 | 10 | 3 | 1 | 1 | 3 | 3 | 2 | 6 | 5 | 4 | 7 | 3 | 3 | 4 | 5 | 1 | 2 | 6 | 5 | 6 | 4 | 9 | | |
| 10 | 34 | 7 | 6 | 5 | 4 | 4 | 4 | 9 | 3 | 3 | 3 | 6 | 6 | 5 | 6 | 4 | 1 | 1 | 3 | 4 | 4 | 6 | 7 | 1 | 5 | 4 | 6 | 12 | 12 | 11 | 9 |
| 11 | 37 | 4 | 3 | 1 | 4 | 3 | 3 | 11 | 1 | 3 | 3 | 1 | 4 | 4 | 3 | 7 | 7 | 5 | 9 | 4 | 4 | 5 | 7 | 3 | 3 | 6 | 4 | 4 | 3 | 9 | |
| 12 | 36 | 6 | 5 | 2 | 2 | 2 | 4 | 11 | 1 | 3 | 1 | 3 | 5 | 4 | 7 | 6 | 9 | 5 | 5 | 5 | 7 | 3 | 4 | 8 | 4 | 4 | 4 | 4 | 9 | | |
| 13 | 35 | 8 | 7 | 5 | 3 | 4 | 2 | 10 | 3 | 6 | 4 | 5 | 1 | 1 | 5 | 5 | 3 | 7 | 2 | 1 | 2 | 5 | 3 | 1 | 8 | 8 | 8 | 7 | 8 | | |
| 14 | 35 | 8 | 7 | 5 | 4 | 4 | 2 | 9 | 2 | 6 | 4 | 5 | 1 | 2 | 6 | 5 | 4 | 7 | 3 | 1 | 1 | 5 | 3 | 2 | 8 | 8 | 8 | 7 | 8 | | |
| 15 | 36 | 5 | 6 | 6 | 4 | 4 | 1 | 10 | 2 | 5 | 3 | 4 | 1 | 7 | 4 | 4 | 2 | 5 | 1 | 2 | 2 | 3 | 1 | 1 | 6 | 8 | 8 | 7 | 9 | | |
| 16 | 34 | 10 | 10 | 8 | 8 | 7 | 4 | 8 | 6 | 6 | 7 | 8 | 5 | 6 | 4 | 1 | 1 | 3 | 4 | 4 | 6 | 7 | 1 | 5 | 4 | 6 | 12 | 12 | 11 | 9 | |
| 17 | 34 | 9 | 10 | 8 | 7 | 7 | 4 | 8 | 5 | 6 | 7 | 7 | 5 | 5 | 4 | 1 | 1 | 3 | 4 | 4 | 5 | 6 | 1 | 5 | 4 | 6 | 11 | 12 | 10 | 9 | |
| 18 | 36 | 10 | 7 | 6 | 4 | 5 | 2 | 11 | 4 | 7 | 5 | 6 | 3 | 4 | 2 | 3 | 3 | 5 | 2 | 4 | 5 | 3 | 4 | 2 | 5 | 10 | 10 | 9 | 10 | | |
| 19 | 37 | 13 | 12 | 12 | 8 | 9 | 5 | 11 | 6 | 9 | 9 | 9 | 7 | 7 | 6 | 4 | 4 | 5 | 5 | 7 | 8 | 4 | 7 | 6 | 6 | 13 | 13 | 11 | 11 | | |
| 20 | 35 | 8 | 7 | 5 | 4 | 4 | 1 | 10 | 3 | 6 | 5 | 5 | 2 | 3 | 1 | 4 | 4 | 2 | 5 | 3 | 3 | 2 | 1 | 6 | 7 | 9 | 7 | 10 | | | |
| 21 | 36 | 8 | 6 | 5 | 5 | 4 | 2 | 11 | 3 | 6 | 4 | 4 | 1 | 1 | 2 | 6 | 5 | 4 | 7 | 3 | 3 | 1 | 7 | 5 | 4 | 10 | 10 | 9 | 8 | | |
| 22 | 35 | 9 | 8 | 6 | 5 | 5 | 2 | 10 | 4 | 7 | 5 | 4 | 2 | 1 | 2 | 5 | 6 | 5 | 8 | 3 | 1 | 8 | 6 | 4 | 9 | 9 | 8 | 7 | 9 | | |
| 23 | 34 | 11 | 10 | 7 | 6 | 7 | 5 | 8 | 6 | 7 | 7 | 6 | 5 | 4 | 1 | 1 | 3 | 4 | 3 | 7 | 5 | 6 | 4 | 6 | 10 | 12 | 10 | 6 | | | |
| 24 | 36 | 7 | 6 | 4 | 3 | 3 | 1 | 11 | 1 | 4 | 3 | 3 | 3 | 2 | 1 | 6 | 5 | 4 | 7 | 2 | 5 | 4 | 5 | 2 | 8 | 7 | 7 | 6 | 10 | | |
| 25 | 36 | 7 | 5 | 4 | 4 | 1 | 10 | 2 | 5 | 3 | 4 | 1 | 2 | 1 | 3 | 4 | 2 | 6 | 1 | 4 | 4 | 4 | 2 | 6 | 8 | 8 | 8 | 7 | 9 | | |
| 26 | 38 | 10 | 8 | 7 | 6 | 7 | 6 | 13 | 6 | 5 | 6 | 8 | 8 | 8 | 6 | 6 | 6 | 5 | 6 | 6 | 10 | 9 | 6 | 8 | 6 | 11 | 12 | 10 | 13 | | |
| 27 | 36 | 3 | 2 | 3 | 6 | 6 | 8 | 15 | 5 | 7 | 4 | 4 | 8 | 7 | 8 | 12 | 11 | 10 | 12 | 8 | 10 | 9 | 10 | 7 | 8 | 11 | 3 | 1 | 7 | | |
| 28 | 35 | 3 | 1 | 3 | 7 | 7 | 8 | 15 | 6 | 8 | 4 | 4 | 8 | 7 | 8 | 11 | 12 | 10 | 13 | 9 | 10 | 9 | 12 | 7 | 8 | 12 | 3 | 1 | 7 | | |
| 29 | 35 | 1 | 1 | 2 | 6 | 5 | 7 | 14 | 4 | 6 | 3 | 3 | 7 | 7 | 7 | 11 | 10 | 9 | 12 | 7 | 9 | 8 | 11 | 6 | 7 | 10 | 1 | 1 | 6 | | |
| 30 | 35 | 4 | 5 | 8 | 10 | 10 | 9 | 8 | 9 | 9 | 9 | 9 | 8 | 8 | 9 | 9 | 9 | 10 | 11 | 10 | 8 | 7 | 9 | 10 | 9 | 13 | 7 | 7 | 6 | | |

It is also known that the company owns two types of trucks available for carrying out the deliveries: vans, with a capacity of 1,000 kg and pickup trucks with a capacity of 1,500 kg.

4. Software implementation

The set of equations (1-9) described in Section 2 was implemented in the commercial software package IBM ILOG CPLEX Optimization Studio release 12.8 for Windows, to solve the proposed problem and minimize the total travel time of the vehicles. Following the structure of the software package used for the solution, the model consists of two different files. The first one is the set of input data of the problem, which for the sake of brevity will not be presented in full in the paper; the second file contains instead the parametric formulation of the problem.

The following figures show the implementation of the problem using the commercial software IBM ILOG CPLEX Optimization Studio. To be more precise, in the first part of the second file (Figure 2), all the variables needed for the formulation of the problem were specified; in the second part of the same file (Figure 3), instead, the formulation of the CVRPTW model is reported.

```

CVRPTW_1.mod
1 |*****
2 | * OPL 12.8.0.0 Model
3 | * Author: Sara
4 |*****/
5
6
7 //Customers
8 int NbCustomers=...;
9 //range Customers=1..NbCustomers;
10 range CustomersAndDepot=1..(NbCustomers+1);
11 //Vehicles
12 int v=...;
13 range Vehicles= 1..v;
14 //Vehicles Capacity
15 int VehicleCapacity = ...;
16 //Demand
17 float Demand[CustomersAndDepot]=...;
18 //TW
19 int LBTW[CustomersAndDepot]=...;
20 int UBTW[CustomersAndDepot]=...;
21
22 //TravelTime
23
24 int TravelTime[CustomersAndDepot][CustomersAndDepot]=...;
25 int ServiceTime[CustomersAndDepot]=...;
26
27 int MinValue = min (i in CustomersAndDepot) LBTW[i];
28 int MaxValue = max (i in CustomersAndDepot) UBTW[i];
29 int M =29;
30
31
32 //Decision Variables
33 dvar boolean x[Vehicles][CustomersAndDepot][CustomersAndDepot];
34 dvar int t[Vehicles][CustomersAndDepot] in MinValue..MaxValue;
35
36
37
    
```

Figure 2. Problem variable – IBM ILOG CPLEX.

```

38 minimize sum (k in Vehicles, i, j in CustomersAndDepot:i!=j) TravelTime[i][j]*x[k][i][j];
39 subject to {
40 //conservazione flusso
41 forall (i in CustomersAndDepot, k in Vehicles)
42 sum(j in CustomersAndDepot:j!=i)x[k][i][j]-sum(j in CustomersAndDepot:j=i)x[k][i][j]=0;
43 //usare al massimo v veicoli
44 forall ( k in Vehicles, j in CustomersAndDepot)
45 sum(i in CustomersAndDepot)x[k][i][j]<=v;
46 //ogni cliente visitato esattamente una volta
47 forall(i in CustomersAndDepot)
48 sum(k in Vehicles,j in CustomersAndDepot:j!=i)x[k][i][j]==1;
49 forall (j in CustomersAndDepot)
50 sum(k in Vehicles, i in CustomersAndDepot:i!=j)x[k][j][i]==1;
51 //ogni veicolo deve partire dal deposito
52 forall (k in Vehicles)
53 sum (j in CustomersAndDepot: j!=1) x[k][1][j]==1;
54 //ogni veicolo deve tornare al deposito
55 forall (k in Vehicles)
56 sum (i in CustomersAndDepot:i!=1)x[k][i][1]==1;
57 // domande minore o uguale della capacita del veicolo
58 forall (k in Vehicles)
59 sum(i in CustomersAndDepot, j in CustomersAndDepot:i!=j)(Demand[i]*x[k][i][j])<=VehicleCapacity;
60 //variabile x[i][i] non definita
61 forall (i in CustomersAndDepot, k in Vehicles)
62 sum (j in CustomersAndDepot, k in Vehicles) x[k][i][i]==0;
63 //rispetto TW
64 forall(i in CustomersAndDepot, k in Vehicles)
65 LBTW[i]<=t[k][i];
66 forall(i in CustomersAndDepot, k in Vehicles)
67 t[k][i]<=UBTW[i];
68 //tempo arrivo
69 forall (i, j in CustomersAndDepot:i!=j, k in Vehicles)
70 t[k][j]>=TravelTime[i][j]+t[k][i]+ServiceTime[i]-M*(1-x[k][i][j]);
71 //tempo partenza deposito
72 forall (k in Vehicles)
73 t[k][1]==0;
74 //tempo arrivo nei nodi
75 forall (k in Vehicles, i in CustomersAndDepot)
76 t[k][i]>=0;
77
78
79
80
81
    
```

Figure 3. Model formulation – IBM ILOG CPLEX.

To apply the model described in the previous sections, some input data were collected, by means of a direct examination of Company A. The full list of input data is provided in Table 4.

Table 4. Input data.

| Parameter | Numerical Value | Measurement Unit | Source |
|----------------|-----------------|------------------|-----------|
| v | 3.00 | - | Company A |
| q | 1,000.00 | Kg | Company A |
| i | 2,...,30 | - | Company A |
| t _s | 5.00 | Min | Company A |

5. Results and discussion

The main results of the simulation runs are reported in the form of a graph. As can be seen from Figure 4, this consists of all nodes of the problem including the depot (node 1). From this node, three different routes depict the paths of each vehicle with their assigned nodes. Moreover, the nodes are connected by arrows with an indication of the time, expressed in minutes, needed to move from one node to another.

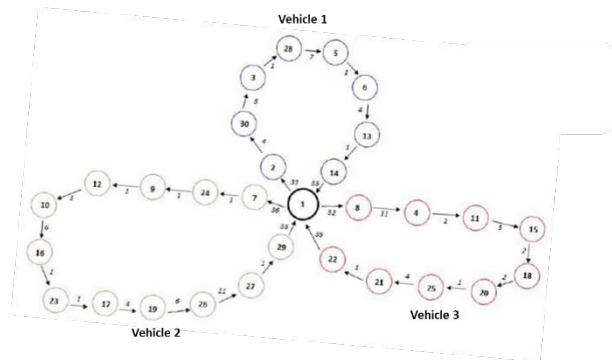


Figure 4. Graph of the routes.

From an analysis of the graph, it is easy to derive the length of the paths travelled by the three vehicles and, consequently, the total travel time, neglecting the service time of each customer, set to 5 minutes.

Table 5. Path's duration.

| Vehicle | Travel Time |
|---------|-------------|
| 1 | 91 min |
| 2 | 107 min |
| 3 | 92 min |
| Total | 290 min |

From the routes, it is also possible to determine the arrival time of each vehicle at the assigned nodes and check if it meets the TW constraint. Adding the travel times of each node to the opening time of the TW ([10:00]), the arrival times at all customers can be obtained. In addition, to determine the departure time from the current node, the value of the service time must be added to the arrival time. Tables 6-8 below show the best sequencing obtained by applying the CVRPTW presented.

Table 6. Results for the first vehicle.

| Node | Travel Time [min] | Service Time [min] | Arrival Time [h] | Departure Time [h] | Tw $[a_i, b_i]$ |
|------|-------------------|--------------------|------------------|--------------------|-----------------|
| 1 | - | - | - | 10:00 | [10:00;18:00] |
| 2 | 33.00 | 5.00 | 10:33 | 10:38 | [10:00;18:00] |
| 30 | 4.00 | 5.00 | 10:42 | 10:47 | [10:00;18:00] |
| 3 | 5.00 | 5.00 | 10:52 | 10:57 | [10:00;18:00] |
| 28 | 1.00 | 5.00 | 10:58 | 11:03 | [10:00;13:00] |
| 5 | 7.00 | 5.00 | 11:10 | 11:15 | [10:00;18:00] |
| 6 | 1.00 | 5.00 | 11:16 | 11:21 | [10:00;18:00] |
| 13 | 4.00 | 5.00 | 11:25 | 11:30 | [10:00;12:00] |
| 14 | 1.00 | 5.00 | 11:31 | 11:36 | [10:00;18:00] |
| 1 | 35.00 | 5.00 | 12:11 | 12:16 | [10:00;18:00] |

Table 7. Results for the second vehicle.

| Node | Travel Time [min] | Service Time [min] | Arrival Time [h] | Departure Time [h] | Tw $[a_i, b_i]$ |
|------|-------------------|--------------------|------------------|--------------------|-----------------|
| 1 | - | - | - | 10:00 | [10:00;18:00] |
| 7 | 36.00 | 5.00 | 10:36 | 10:41 | [10:00;18:00] |
| 24 | 1.00 | 5.00 | 10:42 | 10:47 | [10:00;18:00] |
| 9 | 1.00 | 5.00 | 10:48 | 10:53 | [10:00;18:00] |
| 12 | 1.00 | 5.00 | 10:54 | 10:59 | [10:00;13:00] |
| 10 | 3.00 | 5.00 | 11:02 | 11:07 | [10:00;18:00] |
| 16 | 6.00 | 5.00 | 11:13 | 11:18 | [10:00;18:00] |
| 23 | 1.00 | 5.00 | 11:19 | 11:24 | [10:00;12:00] |
| 17 | 1.00 | 5.00 | 11:25 | 11:30 | [10:00;18:00] |
| 19 | 4.00 | 5.00 | 11:34 | 11:39 | [10:00;18:00] |
| 26 | 6.00 | 5.00 | 11:45 | 11:50 | [10:00;18:00] |
| 27 | 11.00 | 5.00 | 12:01 | 12:06 | [10:00;18:00] |
| 29 | 1.00 | 5.00 | 12:07 | 12:12 | [10:00;14:00] |
| 1 | 35.00 | 5.00 | 12:47 | 12:52 | [10:00;18:00] |

Table 8. Results for the third vehicle.

| Node | Travel Time [min] | Service Time [min] | Arrival Time [h] | Departure Time [h] | Tw $[a_i, b_i]$ |
|------|-------------------|--------------------|------------------|--------------------|-----------------|
| 1 | - | - | - | 10:00 | [10:00;18:00] |
| 8 | 32.00 | 5.00 | 10:32 | 10:37 | [10:00;18:00] |
| 4 | 11.00 | 5.00 | 10:48 | 10:53 | [10:00;18:00] |
| 11 | 1.00 | 5.00 | 10:54 | 10:59 | [10:00;18:00] |
| 15 | 3.00 | 5.00 | 11:02 | 11:07 | [10:00;18:00] |
| 18 | 2.00 | 5.00 | 11:09 | 11:14 | [10:00;18:00] |
| 20 | 2.00 | 5.00 | 11:16 | 11:21 | [10:00;18:00] |
| 25 | 1.00 | 5.00 | 11:22 | 11:27 | [10:00;18:00] |
| 21 | 4.00 | 5.00 | 11:31 | 11:36 | [10:00;18:00] |
| 22 | 1.00 | 5.00 | 11:37 | 11:42 | [10:00;18:00] |
| 1 | 35.00 | 5.00 | 12:17 | 12:22 | [10:00;18:00] |

The outcomes in Tables 6–8 show that all TWs are respected. In particular, the amount of customers

served accounts for 8 with 2 hours and 16 minutes of travel using the first vehicle; 12 with 2 hours and 52 minutes of travel using the second one; 9 with 2 hours and 22 minutes using the third one.

6. Conclusions

The proposed method aims to solve the CVRPTW and identify the most desirable solutions. The objective is to minimize the transportation time while satisfying vehicle's capacity constraints and customer's demand. This paper has proposed an analysis of the efficiency of the delivery considering the travel time by the CVRPTW model of a real company of Southern Italy operating as a CEP service provider (Company A), consisting of a depot, three vehicles and several customers. The model was developed with IBM ILOG CPLEX in a simulation optimization procedure, which took into account the travel time of vehicles and the demands of customers.

From a theoretical perspective, the model developed in this paper includes a set of formulae that were embodied in an IBM ILOG CPLEX software, consisting of 2 files (one for input data, the other for CVRPTW formulation), for the computational of the travel time of the system. From a practical perspective, this paper is structured as a case study, as it focuses on the specific context of Company A. For testing purpose, the model was applied to evaluate the performance of the company under examination, whose input data were obtained from a data collection phase involving the same company previously interviewed to develop the model. An example of practical use of the outcomes generated by the model was also proposed. From a technical perspective, the development of an evaluation model to optimize the performance of a CEP service provider represent an interesting addition to the literature, both because the assessment of performance is not a trivial task and because e-commerce is recognized as a key area for improvement.

Some limitations of the study should also be mentioned. A general weakness of the model is that it is hard to test the efficiency of the results provided. In the case under examination, none of the managers of CEP service provider involved in the analysis had carried out a (previous) personal evaluation of the performance of the company's distribution activities. Moreover, Company A could not provide the routes effectively travelled that 28 February 2019, because of confidentiality reasons; according to that it was not possible to compare the simulated ones with what really happened in the company. On the contrary, relative analyses, such as comparisons of the performance of different CEP service provider or of different customers of the same CEP service provider, could be done in the future and could provide useful guidelines for developing a performance improvement strategy.

More in general, the model developed in this paper

can be taken as the starting point for further evaluations in other companies and can be adapted to different scenarios from the CEP provider. When analyzing different systems, it could be interesting to evaluate whether greater customers' demands could modify the performance of the system or modify the strategy to be adopted for delivery management. Another future development refers to the vehicle routing model used. In particular, it could be very interesting to consider the demand of customers during the day considered in this study.

By properly modifying the objective function and constraints, the model could also be implemented for determining the exact number of vehicles required to accomplish a given set of routes taking into account the time windows of the delivery.

Finally, the model itself represents an interesting addition to the literature about the CVRPTW in the CEP field.

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