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Sheet metal assortment optimization with k-Means

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Abstract

This paper deals with inventory optimization in the procurement of sheet metal for small series and contract manufacturers. An AI-based algorithm was developed and tested for a real-world application. The underlying use case is a variant of the cutting stock problem. The objective is to optimize the number of metal sheet variants to be source to reduce inventory and handling costs. Therefore, demand for metal sheets with different dimensions are combined so that large sheets are sourced from which the sheets required for production are cut. Here we have a trade-off between the savings in inventory management versus the additional costs due to scrapping of cut losses. Decision models for this application case which take all these factors into account have not been developed so far. This paper presents a first approach by modelling parts of the problem as a linear integer problem which is heuristically solved with a k-Means clustering algorithm. With this model different scenarios can be simulated for making better inventory decisions. This approach was implemented in a software application based on Power BI and the Python library scikit-learn and tested in practice by a company. The test showed considerable savings potential for the use case and provided important stimuli for further research, especially regarding further modelling approaches and search algorithms.

Keywords: assortment planning, cutting stock problem, machinery construction, artificial intelligence

1. Introduction

Manufacturing companies generate their products and services through using production resources such as machines and labor and sourcing parts and materials from their suppliers. So, the sourcing of materials is a necessary activity to operate an enterprise. Here, there is a well-known conflict of objectives about the optimal inventory level. Having too much inventory binds capital and requires warehousing capacity. Operating business with low inventory levels will result in lower customer services due to lacking material. This has been researched for nearly a century in the field of operations research and production management. This is especially applicable for the sourcing for industries like the automotive sector or manufacturing industries that are characterized by assembling products from parts.

However, the procurement of steel and metal from in form of sheets, rods, and coils is different from the discrete parts sourcing. Materials can be substituted too a far greater extent than in other industries. This offers the opportunity for inventory reduction through combining substitutable materials. That substitution mostly happens through sourcing a large sheet type from which different sheets are cutting according to the production demand. In addition, material warehousing is different in such that sheets – similar arguments are true for rods and coils – are typically stored in stacks on the ground. In general, the material range is so large that these stacks have different material types. A storage in compartments with random access to each type of material is often not economically feasible -especially if these sheets to be sourced weighs more than a ton and are best handled by roof mounted gantry cranes. Furthermore, it is the nature of these stacks, that one must move the sheets lying on-top of the required sheet before this can be taken from the warehouse for the subsequent manufacturing processes. In addition, the steel and metals sector is characterized by production in large lot-sizes and this demand frequently comparable large minimal order batch sizes of 20 or more tons. And finally, steel and metal materials can be comparable easily by recycled. There is a well-established ecosystem for collecting and reusing scrap. However, sourcing sheets that will be partially scrapped is associated with economic losses.

In summary, the procurement of sheets is thus a non-trivial multi-factorial decision making problem. One important aspect of this application case is that the inventory levels must be optimized to meet future demand. However, this demand is not deterministic and difficult to forecast for the required time horizon. This is due to two reasons. First, the number of different materials is considerable large in respect to the demand for the different sheets. For the very most of the sheets to be purchased the annual demand comes from only a few production orders. Often more than 80% of purchase volume is associated with sheet that have a sporadic demand pattern. Second, the delivery times of the sheet suppliers is usually several months and thus far to long that sheet can only be purchased based on customer orders. Because of these reasons it is suitable to perform the inventory optimization for different demand cases, thus simulating possible future scenarios.

Surprisingly, this application area has not yet been sufficiently covered by scientific research. A SCOPUS query from July 2022 searching for articles that contain the words inventory, policy, and steel in the title, abstract or key words yielded only 134 hits, none of which were fitting to the application case of this paper. Consequently, we have broaden the literature review to include application cases which have similar characteristics as the procurement of materials from the metals and steel industry.

2. State of the Art and Related Research

The focus of this research was to develop a better solution for a real-world application case. Most manufacturing companies apply very established methods for their inventory management which are part of their ERP software systems and can be found in many textbooks. One prominent method is the ABC-Analysis. It divides a set of objects into classes A, B and C, which are ordered by descending importance such as turnover or demand frequency (see also Chen et al. 2008). However, the ABC-Analysis is very coarse-grained method and an investigation of modern Operations Research is advised. For the state-of-the-art analysis, we have focused on two main areas: The assortment problem and the cutting stock problem.

Assortment strategies occur when a company is unable to have all materials applicable to its business in stock and thus must choose of what to store and what not. This problem is frequent in the retail sector but can also apply to sourcing of materials. A comprehensive review of the scientific literature to the assortment or catalog problem was given in 2008 by Pentico. He structured the literature according to features such as demand pattern, dimensions, number of stock sizes, possibilities to substitution and costs. A more recent overview of retail analytics which includes the assortment planning was given by Sachs in 2015. At a first glance the application case of sheet metal assortment has much in common with the assortment planning in the retail sector. But these models assume that a sourced part is substituted by another existing part. That means, that these parts are distinct, fixed units and not materials that can sourced in different geometrical form.

In that context, Discrete-Event-Simulation (DES) has been applied by several authors for evaluating different inventory policies in respect to costs and part availability. Hafner et al. 2019, for instance, have build a DES system to investigate the effects of spare part inventory pooling when several companies share spare parts. Teter et al. 2019 developed a simulation model to explore the effects of different concepts of operations for inventory management of the US Naval Supply Systems Command Weapon Systems Support (NAVSUP WSS). In particular, they applied DES to investigate the transition time needed when switching between inventory policies. Due to changing demand distributions, NAVSUP WSS quarterly optimizes its inventory policies of approximately 375,000 line items. The inventory of these line items is managed by an (s,Q) policy and an the inventory parameter of reorder point s and order quantity Q are periodically optimized with an mixed-integer linear program. The simulation has shown, that it takes up to 6 month before the effects of changing an inventory policy become evident.

A better fitting research area covers the cutting-stock problem. This application case is the cutting standard-sized pieces of stock material, such as sheet metal, paper rolls, cardboard sheets, or wood sheets, into pieces of required sizes while minimizing waste (Wäscher et al., 2007). A difference to the application case presented here, however, is that the with the cutting stock problem it is possible to cut several sheets from a single sourced sheet. In such a way, the cutting stock problem has much in common with the knapsack problem (see also Horowitz & Sahni, 1974).

Nevertheless, approaches to solve the cutting stock problem appear to be best suited for our application case. Here especially search and clustering algorithms appear to be most promising (Vahrenkamp, 1996; Prestwich et al., 2015; Mostafa & Eltawil, 2017; Garcia et al. 2018). In recent years the research group at the University of Modena and Reggio Emilia has also published algorithms and benchmarks for the cutting stock optimization problem. This group published a survey of exact solution techniques for two-dimensional cutting and packing problems (Iori et al., 2021) as well as 2DPackLib library, a library of different packing algorithms and benchmarks (Iori et al., 2022).

Another industry facing the same problems as presented in the metal sheet application case is the manufacturing of paper. Paper mills produce a variety of different paper rolls that vary in paper quality, thickness, width of the paper roll, and run length (Chauhan et al., 2008). The manufactured rolls are slitted into smaller rolls (lengthwise divisions) according to customer demand. Thereby paper waste occurs if the customer demand cannot be matched lossless to the manufactured paper roll. This slitting loss can be reduced by increasing the number of production variants. In a further step converting plants source paper roll to cut these into sheets. Again, we have conflicting goals between reducing cutting loss and reducing paper mill inventory. A recent article for this application case presented an arc-flow formulation for solving paper-cutting stock problems (cf. Mehdi et al., 2021).

Our literature analysis has shown that our application case has much in common with bin-packing and cutting problems. However, there is not an exact fit to the published application areas. This might be due to the special requirements of the considered manufacturer. Nevertheless, the utilization of integer-programming approaches and – due to the problem size – heuristic solving algorithms appear to be an approach for reducing the number of sourced metal sheets.

3. Description of the Application Case

3.1. Decision Problem

Many companies in plant and machine construction procure metal sheets for the manufacture of their products. These sheets differ in terms of material and surface finish as well as thickness, width, and length. Many different sheets are used, especially in project and small series production. Often also in very small batch sizes. On the other hand, the companies cannot economically procure these very specific sheets in a quantity that corresponds to the requirements for the respective orders. Instead, the mechanical engineering companies must purchase larger lot sizes from the metal and steel manufacturers. This results in a numerous sheet metal variants – In the application case of this article, we are talking about several thousand different sheets -that the company must store for many months.

The high number of sheet metal variants to be stored leads to two problems: First, there is the net working capital. Due to small batch production, surplus sheet metal often must be stored for a very long time before it can be used for a new customer order. Secondly, many variants cause high handling costs. It is not economically feasible to store thousands of variants in single-variety stacks - the storage space required would be far too high. Instead, they are stored in mixed stacks. To access a sheet, many other sheets are therefore moved in practice to get to a specific sheet variant (cf. figure 1).

Against this background, an approach to reduce the number of sheet metal variants was developed for this work. Sourcing demand for sheets that have the same metal quality, surface treatment and thickness can be combined. If, for example, two sheets with a length of 10 meters and a width of 1.5 meters are required for one customer order and three 8 x 1.8 sheets are required for another customer order, the number of sheet variants can be reduced as follows. The company sources sheets with the dimensions of 10 x 1.8 from its sheet supplier and cuts the not needed sections. This approach has both advantages and disadvantages. On the plus side reduction of the number of sheet variants leads to less bound capital and reduced handling effort to access specific sheets. Furthermore, administrative costs for ordering sheets could be reduced because the total number sourcing orders drops. Then again, the cut metal from the large sheets is scraped.

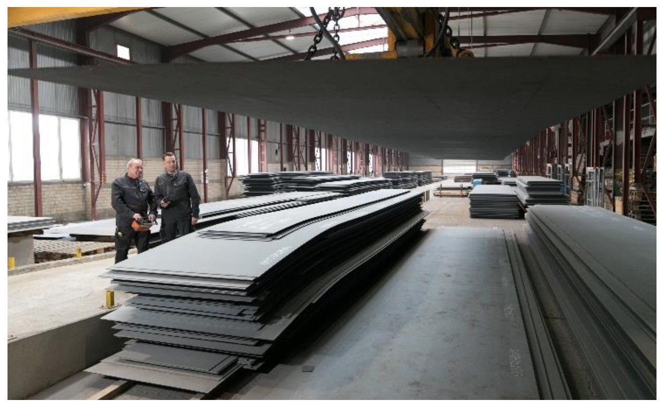

Figure 1. Sheet metal warehouse at the application case

It is not a trivial task to determine which different sheets should be combined. Hence, we have developed an AI-based clustering algorithm for this application case.

3.2. Problem Formalization

In figure 2 an application case with 12 sheets that differ in width and length but are same in quality, surface, and thickness, is shown. The size of the circles in this diagram relates to the annual demand. For this scenario the sheets are clustered in two three groups. Note that cluster 2 consists of three different sheets of which one has both the greatest length and width of.

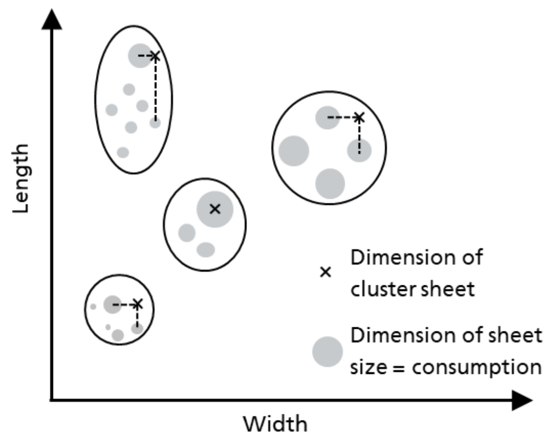

Figure 2. Clustering 19 sheets into 4 clusters

But which kind of clustering is optimal based on the different sheet sizes, the customer demand and costs factors for bound capital, warehousing handling, scrap losses and ordering costs? This research question can be modeled as integer linear program (Schrijver, 1998). For the sake of simplicity, we assume for the time being that the number of clusters is also given. We will address this determination of the optimal number of clusters in section 4. We have the following input variables to describe the cluster:

$C = the\ set\ of\ all\ clusters$

$k = number\ of\ clusters$

$c_j = a\ specific\ cluster; 1 \leq j \leq k$

The sheets are specified as follows:

$S = the\ set\ of\ all\ sheets$

$n = number\ of\ sheets$

$s_i = a\ specific\ sheet; 1 \leq i \leq n$

$sl_i = length\ of\ sheet\ s_i\ in\ meters; 0 < sl_i$

$sw_i = width\ of\ sheet\ s_i\ in\ meters; 0 < sw_i$

Each sheet is assigned to exactly one cluster. Therefore, the relationship between clusters and sheets can be described as follows:

$mc_j = number\ of\ sheets\ in\ cluster\ c_j; 1 \leq mc_j \leq n$

$c_{j_m} = sheet\ m\ of\ cluster\ c_j; 1 \leq m \leq mc_j$

For each cluster only one sheet will be sourced from the suppliers and stored at the warehouse.

$cs_j = required\ sheet\ of\ cluster\ c_j$

$csl_j = length\ of\ required\ sheet\ of\ cluster\ c_j$

$csw_j = width\ of\ required\ sheet\ of\ cluster\ c_j$

$cst_j = thickness\ of\ required\ sheet\ of\ cluster\ c_j$

The width and length of this sheet must be the respective maximum of all sheets of this cluster. Note that it is possible that one of the sheets of a cluster is the required sheet. However, figure 2 also illustrates the case where the required sheet is not of the sheets of the cluster. Note further, that only the required sheet of a cluster is sourced and stored. When a customer demand for a sheet of this cluster occurs, then the required sheet is cut to the required dimensions. Consequently, we have costs which relates to the offcut area.

The optimization problem is to allocate the sheets to the given clusters and determine the length and width of the required sheets of these clusters so that cutting losses are minimized. The number of possible allocations depends on the number of clusters and the number of sheets:

$number\ of\ allocations = k^n$

Note, that the costs for allocating a single sheet to a cluster can only be computed when all sheets to this cluster are assigned. We model the assignment of a sheet s_i to a cluster c_j with a binary relation $r_{i,j}$.

$r_{i,j} \in \{0,1\}$

The optimization problem can thus be described as an integer program with the following minimization function:

$$min \sum_{i=1}^{n} \sum_{j=1}^{k} r_{i,j}(csl_j \times csw_j - sl_i \times sw_i)$$

The length and width of the required sheet cs_j for a cluster c_j is computed as follows:

$csl_j = \max(sl_i)\ with\ 1 \leq j \leq k, 1 \leq i \leq n\ and\ r_{i,j} = 1$

$cwl_j = \max(sw_i)\ with\ 1 \leq j \leq k, 1 \leq i \leq n\ and\ r_{i,j} = 1$

Furthermore, each sheet is assigned to exactly one cluster:

$$\sum_{j=1}^{k} r_{i,j} = 1\ with\ 1 \leq i \leq n$$

Through modeling the application case as an integer linear program, it is possible to compute the optimal solution with a brute-force approach by trying all possible allocations. However, this type of problem belongs to Karp's 21 NP-complete problems (Karp, 1972) that are not computable for practical applications.

4. Using k-Means for Sheet Metal Assortment

4.1. Data basis of the application case

The developed solution aims to reduce the number of sheet variants sourced from all suppliers of the manufacturing company. A first step of the preprocessing of the imported data was to eliminate sheet metals that did not had any demand in the also imported transaction data. Then the different metal sheets were grouped into product groups.

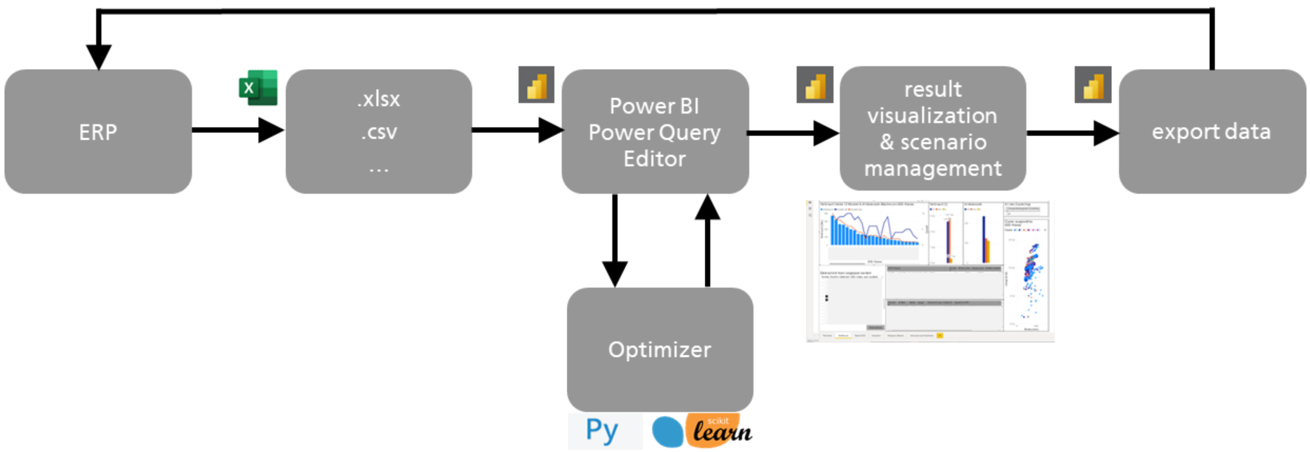

Figure 3. Architecture of the developed software system for the sheet metal assortment optimization with k-Means

The assortment to be optimized consists of product groups, which differ in sheet thickness, surface quality and material. The sheet thickness ranges from 4 mm to 20 mm.

There are around 50 different material types, e.g., 1.4571 / X6CrNiMoTi17-12-2 and about 10 different surface types, e.g., sandblasted and primed. The combination of demand for sheets is only possible for the sheets of the product group. For the application case we had in total a demand for more than 2000 different sheets and about 150 different product groups.

4.2. System Overview

For this research, a clustering approach was developed based on a real-world application case, that is, for a specific company and its range of sourced metal sheets. We used demand data from 2021 as a model for the future demand. Furthermore, master data on customers, sourced metal sheets, and suppliers have been used. This data is exported from the ERP system of the company through an Excel or CSV file. An Excel file is imported into in our application case into a Power BI solution (Becker & Gould 2019), preprocessed and feed to the scikit-learn library (Pedregosa et al. 2011) to call a k-Means algorithm. The results a presented with a Power BI dashboard (cf. figure 3).

4.3. Sheet Clustering for a Product Group

The sheet clustering is performed with a k-Means algorithm (Benabdellah et al., 2019) for each product group. Note, however, that we compute the optimal clustering only for a given number of clusters. The reason for this approach was to limit computation time. On average we have around 15 sheets in a product group. So it would be necessary to compute clustering with 2 to 14 clusters – 1 cluster and 15 clusters are the trivial cases. However, the number of clusters per product group is set based on a target value for the total number of sheets to stored. This heuristic approach is justified from the constraints of the application case, in particular the number of different sheet variants that can be stored in the existing warehouse. In the past, the company required additional external warehousing capacities.

The k-Means clustering for a product group used the following input parameters:

$k = number\ of\ clusters$

$c_j = a\ specific\ cluster; 1 \leq j \ \leq k$

$n = number\ of\ sheets$

$s_i = a\ specific\ sheet; 1 \leq i \ \leq n$

The k-Means clustering algorithm works as follows:

1. Randomly select for each cluster c_j a sheet s_i as the center (the means) of this cluster.
2. Assign each of the remaining sheets to the cluster with the means that is closest.
3. Compute the means of each cluster

Steps 2. and 3. are iterated until the allocation of the sheets to the clusters is stable.

The k-Means implementation of the scikit-Learn library that was used for this application case modifies the algorithm in two ways (cf. Arthur & Vassilvitskii, 2006). First, the selection of start values (seeds) for the cluster is performed tenfold (10 populations). Second, the algorithm stops latest after 300 iterations. The population with the lowest distances of the sheets from their respective clusters is selected.

As a final step of our clustering algorithm the length and width for the representative sheet cs_j for each cluster is computed as the respective maximum of the lengths and widths of all sheets allocated to this cluster.

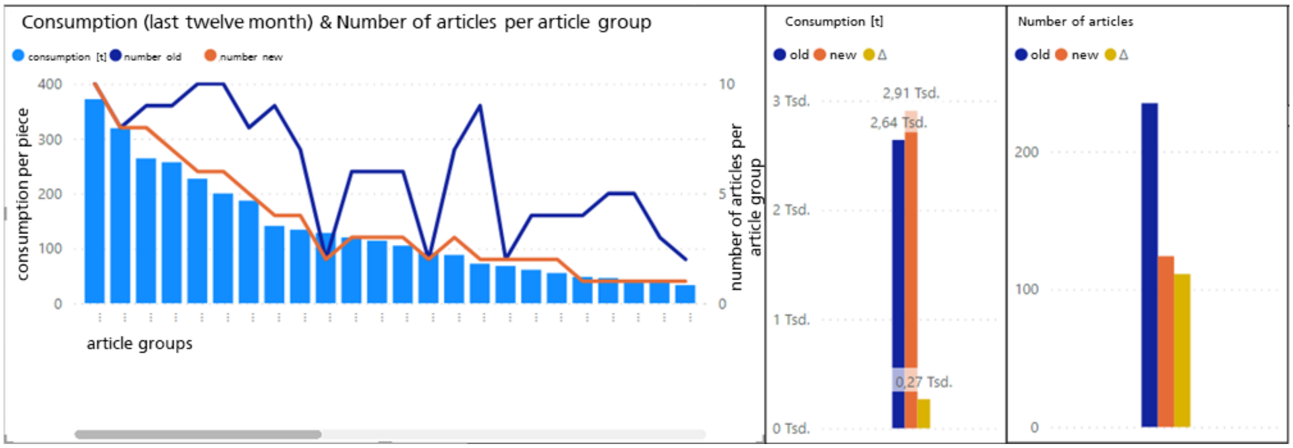

Figure 4. Excerpt of the dashboard of the developed application

4.4. Evaluation and visualization of the sheet clustering

The optimization performed with the k-means algorithm was solely based on the surface area of the sheets. However, for the application case it was necessary to compute effects of different clustering scenarios to scrap loss, ordering costs, and average inventory levels. An Economic-Order-Quantity (EOQ) policy was used to calculate the effects of ordering costs and inventory level. The scrap loss for each sheet is computed in the following way:

- Starting point is the demand from the customer order and the allocation of the sheets to the clusters
- For each sheet s_i the cut loss volume is computed as the difference of the length of the required sheet cs_j minus the length of the s_i sheet. In the same way the width difference to the required sheet is computed.
- The cut loss volume is then the product of these differences multiplied by the thickness $cst_j$of the require sheet. This volume is then multiplied by a density factor that was selected as $7.85\frac{t}{m^3}$. The cut loss for a single sheet in weight is this the result.
- Then the scrap loss – the cut metal can be sold as scrap, but this price is lower than the purchase price for the sheet – is computed based on product group specific purchase prices and a loss percentage.
- Finally, the cut loss for a single sheet is multiplied with the demand of this sheet.

This computation is performed for all sheets of all product groups. Subsequently, the results of the clustering and evaluation are shown in a dashboard (cf. figure 4). A key finding from the evaluation with Employees from Ferro Umformtechnik was that the visualization widgets are well suited to explain the solutions found to the users. In figure 4, for instance, we can see on the left side a diagram showing (sorted according to consumption volume from left to right) the 24 first product groups with their respective annual sheet consumption (blue columns), the number of sheet variants without clustering (dark blue line) and with clustering (orange line). In the same fashion the column diagrams on the right show the increase in consumption and the decrease in the number of sheet variants.

5. Results and Discussion

The objective of this research was the development and evaluation of an application-specific artificial intelligence model for a real-world case. Consequently, we pursued an iterative development approach with a permanent end-user involvement. This gave us considerable insights into requirements form a business perspective.

Here the combination of an easy to customize analysis tool like Power BI with the algorithmic sophistication of open-source AI-libraries like scikit-learn enabled us to rapidly implement the solution and constantly realized new functions for the tool. One notable example was the requirement to perform the clustering based only on the metal sheet demand for a single customer. This additional functionality could be quickly realized through standard features of Power BI. As a result, the company is using the solution to reduce the number of its sourced sheets.

The work presented in this paper is a first step towards an application-oriented approach for inventory management of sheet metals in manufacturing for enterprise that purchase a large variety of materials. The model performs an optimization on geometry and material costs – based on customer-specific purchasing rates for the different sheets and suppliers. However, the effects of combining sheets on handling, warehousing, and inventory levels are not yet part of the optimization model. Instead, the user has to perform several optimization runs for different target numbers of clusters and/or different demand scenarios. Thus, the developed solution shows the basic applicability. Ferro Umformtechnik is using the tool since the beginning of 2022 to simulate the effects of different demand patterns and different target values for the total number of its sheet variants on its operation. Based on these scenarios, Ferro Umformtechnik is reducing the current number of its sourced metal sheets and determine warehousing capacity needed in the future. The toolset supports this decision making by calculating number of sheet variants, inventory levels and associated costs for future demand scenarios.

6. Conclusions and Outlook

The application of the developed tool for the operational decision making by Ferro Umformtechnik showed the utility of the of the modelling approach. The combination of dashboard functionality of modern business intelligence solutions with the algorithmic capabilities of open-source AI libraries shows potentials for application in manufacturing ad supply chain management. The technology has reached a maturity for real-world application, but the experiences gained with the use case of a single manufacturing company clearly indicates the need for further improvement.

Based on funding from the Center of Logistics and IT – an ecosystem combining several research institutions form Dortmund, Germany – we are elaborating both the modelling approach and the tool implementation. The tool implementation will benefit from further improvements: The interface of Power BI with python is difficult to debug and consumes considerable time. A clustering run takes from one to several minutes depending on the target number of sheets. That is the major reason why we did not yet have implemented an algorithm that enumerates the number of clusters for all product group to determine both the optimal number of clusters and the optimal allocation of sheets to these clusters.

From a scientific perspective we are working to extend the modelling approach by considering further costs and performance attributes. The handling, warehousing, and sourcing costs are currently only used for the evaluation of a generated sheet clustering solution. However, these factors should be part of the clustering algorithm to better compute the optimal number of clusters. In addition, it would be interesting to simulate different optimization scenarios in a systematic fashion to evaluate the robustness of inventory policies for different demand scenarios. Furthermore, we intend to continue our applied research through improving the clustering algorithm and evaluating other clustering approaches such as DBSCAN or Gaussian Mixture as well as further Operations Research approaches not based on clustering.

Finaly, the assortment problem presented in this paper can also be transferred to various other industries such as the wood, paper, textile, and plastics industries, which have similar decision problems when sourcing raw materials.

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