



# A credibility allocation approach of complex simulation systems

Huan Zhang<sup>1,2</sup>, Wei Li<sup>1,2</sup>, HuaPin Geng<sup>3</sup>, Ping Ma<sup>1,2</sup>, and Ming Yang<sup>1,2,\*</sup>

<sup>1</sup>Control and Simulation Center, Harbin Institute of Technology, Harbin, 150080, China

<sup>2</sup>National Key Laboratory of Modeling and Simulation for Complex Systems, Harbin, 150080, China

<sup>3</sup>Science and Technology on Complex System Control and Intelligent Agent Cooperation Laboratory, Beijing, 100074, China

\*Corresponding author. Email address: frank@hit.edu.cn

## Abstract

Complex simulation systems usually need to satisfy the credibility requirements and the credibility of complex simulation systems is inextricably linked to the credibility of simulation sub-systems. Hence, in order to guarantee the credibility of complex simulation systems meets requirements, it is necessary to allocate the credibility of simulation sub-systems. Complex simulation systems are composed of several correlative simulation sub-systems and their credibility has correlation relationships, which makes it difficult to obtain the credibility of simulation sub-systems quickly. The existing works mostly ignore the relationships between the sub-systems and are difficult to generate the credibility allocation results of simulation sub-systems. In this paper, we model the credibility allocation problem as a probabilistic inference problem based on the pairwise Markov random field (PMRF). In addition, we apply several inference algorithms to perform the inference exactly and approximately in the credibility PMRF. Finally, experiments are conducted on the air defense combat complex simulation system, and the experiment results demonstrate that the proposed method can obtain the credibility allocation results quickly and accurately.

**Keywords:** Complex simulation system; credibility allocation; PMRF; inference algorithms

## 1. Introduction

In the past decades, systems continually grow large and complex. Digital twin systems (Liu et al., 2021), the Internet of autonomous things (Hemmati et al., 2022), and other complex systems have emerged and been widely used. Modeling and Simulation technology provides a solution for the study of complex systems (Mykoniatis and Angelopoulou, 2020). Complex simulation systems play an important role in the design, development, analysis, and testing of complex systems. The credibility of the complex simulation system is a crucial measure to evaluate the complex simulation system, which directly relates to the success or failure of

the complex simulation system (Li et al., 2018). The credibility of the complex simulation system usually needs to meet the requirements given by users of simulation systems. However, complex simulation systems usually adopt the modular development approach and consist of several simulation sub-systems with complex correlations. The credibility of the complex simulation system and simulation sub-systems influence each other, which makes the credibility of simulation sub-systems need to reach certain values to ensure that the complex simulation system meets the credibility requirements. Hence, the credibility allocation values of simulation sub-systems need to be obtained before developing simulation sub-systems and the credibility allocation approach needs to be studied.



Several credibility evaluation approaches of complex simulation systems have been proposed for evaluating credibility from different aspects. They analyze the characteristics of complex simulation systems deeply and perform the evaluation based on their features. However, most existing approaches cannot obtain the credibility of simulation sub-systems given credibility prior information and credibility requirements. Besides, most existing credibility evaluation processes are difficult to explore the relationship between the credibility of simulation sub-systems abundantly, which leads to inaccurate evaluation results. In this paper, we propose a probabilistic model based on the pairwise Markov random field (PMRF) which can show the relationship between the credibility of components clearly and accurately and generate credibility allocation results. We apply the proposed approach to infer the credibility of simulation sub-systems in the air defense combat complex simulation system and discuss the inference performance under different parameters and inference algorithms.

The rest of this paper is organized as follows. Section 2 provides a brief introduction to related work. Section 3 explains our proposed credibility allocation approach of simulation sub-systems within complex simulation systems, including credibility probabilistic inference problem description, credibility PMRF model, and credibility inference algorithms. Section 4 analyzes the experimental results thoroughly, and Section 5 draws conclusions and highlights future work.

## 2. Related Work

With the complexity and scale of complex simulation systems increasing, the credibility evaluation of complex simulation systems is becoming important. However, most traditional credibility evaluation approaches have several disadvantages, such as low computational efficiency, inaccurate evaluation results, etc. The main reason is that these approaches don't take into account the complex interaction between sub-systems, which makes the relationships between credibility ignored.

A number of credibility evaluation approaches utilize probabilistic graphical models to realize the credibility evaluation of complex simulation systems. (Mahadevan et al., 2005) first used a Bayesian network to infer the credibility information of the overall model from the credibility of the sub-modules. By exploiting the structure of Bayesian networks and available experimental observations, the MCMC approach is used to infer the posterior density of the performance function, which is constructed as capacity minus demand, and calculate the credibility of the overall model. (Jiang et al., 2010) modeled the effect of low-level test data on system-level model evaluation as a Bayesian network using structural equation modeling and proposed an interval hypothesis-based Bayesian model validation approach to provide more consistent evaluation results. Based on these ideas, a Bayesian inference approach for model validation and confidence

extrapolation is proposed (Lin et al., 2020). The authors constructed a new Bayesian network that introduces input variables and observed output variables and employed the highest posterior density confidence range to quantify the effect of lower-level data on the system-level model evaluation and improve the accuracy of credibility results. Besides, (Ma and Wu, 2014) proposed an evaluation approach using a two-layer model based on the fuzzy analytic network process. Mission network and capabilities and systems network are applied to represent the complex relationships between the sub-systems under different sub-missions and the evaluation is processed based on these networks utilizing expert knowledge.

Nonetheless, most of the studies in the existing literature focus on inferring the overall credibility from the credibility of the sub-systems and there are few researches on the credibility allocation of the simulation sub-systems. Credibility allocation also faces many challenges, such as uncertainty of the credibility, modeling of complex interaction between sub-systems, fast credibility allocation, and so on. Thus, the credibility allocation problem of the simulation sub-systems which comprise a complex simulation system needs to be tackled. PMRF is a widely used undirected graphical model and it characterizes the prior information of each node and the relationships among neighbors utilizing the unary function and the pairwise potential function (Zhang and Li, 2014). Thus, we tackle the credibility allocation problem of simulation sub-systems within complex simulation systems using PMRF.

## 3. Credibility allocation approach of simulation sub-systems

In order to generate the credibility allocation results of the simulation sub-systems, we formulate the credibility allocation problem of simulation sub-systems within complex simulation systems as a probabilistic inference problem using a PMRF model. And then, the probabilistic inference problem can be carried out by using the inference algorithms of PMRF.

### 3.1. Credibility probabilistic inference problem description

Suppose the complex simulation system  $\mathcal{S}$  consists of a set  $\mathcal{S}$  of  $M$  simulation sub-systems,  $\mathcal{S} = \{S_1, S_2, \dots, S_M\}$  and the credibility of the complex simulation system and simulation sub-systems are  $\mathcal{C}_{sys}$  and  $\mathcal{C}_{sub}$ , where  $\mathcal{C}_{sub} = \{C_{sub,1}, C_{sub,2}, \dots, C_{sub,M}\}$ . Considering the correlation between the credibility of simulation sub-systems, we need to jointly model all credibility variables. Let  $P(\mathcal{C}_{sub}, \mathcal{C}_{sys})$  denote the joint probability distribution of  $\mathcal{C}_{sub}$  and  $\mathcal{C}_{sys}$ . Besides, we model the credibility as a discrete random variable, whose value is from a discrete set  $\Gamma = \{\gamma_1, \gamma_2, \dots, \gamma_N\}$ .

The critical goal in this work is to allocate the credibility value of each simulation sub-system given  $\mathcal{C}_{sys} = \zeta$ ,  $\zeta \in \Gamma$ . To compute the credibility allocation

results of simulation sub-systems, we calculate the joint probability  $P(\mathbb{C}_{sub}, \mathbb{C}_{sys} = \zeta)$  for all possible values of  $\mathbb{C}_{sub}$  and the value  $\mathcal{C}^* = \{c_{sub,1}^*, c_{sub,2}^*, \dots, c_{sub,M}^*\}$ ,  $c_{sub,i}^* \in \Gamma$ ,  $i = 1, 2, \dots, M$  corresponding to the maximum joint probability is the final credibility value of simulation sub-systems. In summary, the credibility allocation problem of simulation sub-systems within complex simulation systems is to find a unique  $\mathcal{C}^* = \{c_{sub,1}^*, c_{sub,2}^*, \dots, c_{sub,M}^*\}$  which maximizes  $P(\mathbb{C}_{sub}, \mathbb{C}_{sys} = \zeta)$ , i.e.,

$$\mathcal{C}^* = \operatorname{argmax} P(\mathbb{C}_{sub}, \mathbb{C}_{sys} = \zeta) \quad (1)$$

However, the computational complexity of Eq. (1) grows exponentially as  $\mathcal{O}(N^M)$  and it is NP-hard. Therefore, we propose the credibility PMRF model that describes the complex joint distribution compactly, and  $\mathcal{C}^*$  can be inferred efficiently utilizing inference algorithms.

### 3.2. Credibility PMRF of the complex simulation system

PMRF is a subclass of MRF where all of the factors are over single variables or pairs of variables, i.e., a set of node potentials and a set of edge potentials (Koller and Friedman, 2009). We model the joint distribution  $P(\mathbb{C}_{sub}, \mathbb{C}_{sys})$  in a credibility PMRF over a graph  $\mathcal{H} = (\mathcal{N}, \mathcal{E})$ , where  $\mathcal{N}$  is a set of nodes and  $\mathcal{E}$  is a set of edges. In the credibility PMRF, the nodes represent the credibility of the complex simulation system and simulation sub-systems and the edges represent the relationships between the simulation sub-systems, as well as between the complex simulation system and simulation sub-systems, which is illustrated in Figure 1. Therefore, the prior knowledge of  $c_{sub,i}$ , the relationship of  $\mathbb{C}_{sub}$  and the relationship of  $\mathbb{C}_{sub}$  and  $\mathbb{C}_{sys}$  are described by the unary potential function  $\phi$ , the pairwise potential function  $\psi$  and the pairwise potential function  $\varphi$  respectively. The joint distribution function  $P(\mathbb{C}_{sub}, \mathbb{C}_{sys})$  is proportional to the product of  $\phi$ ,  $\varphi$ , and  $\psi$ . Then, we can represent  $P(\mathbb{C}_{sub}, \mathbb{C}_{sys})$  as a credibility PMRF  $\mathcal{H}$  as follows

$$\begin{aligned} \tilde{P}(\mathbb{C}_{sub}, \mathbb{C}_{sys}) &= \prod_{\alpha} \phi_{\alpha}(c_{sub,\alpha}) \prod_{\beta} \varphi_{\beta}(c_{sub,\beta}, c_{sys}) \prod_{(\eta,\gamma) \in \mathcal{E}} \psi_{\eta\gamma}(c_{sub,\eta}, c_{sub,\gamma}) \\ Z &= \sum_{\mathbb{C}_{sub}, \mathbb{C}_{sys}} \tilde{P}(\mathbb{C}_{sub}, \mathbb{C}_{sys}) \end{aligned} \quad (2)$$

$$P(\mathbb{C}_{sub}, \mathbb{C}_{sys}) = \frac{1}{Z} \tilde{P}(\mathbb{C}_{sub}, \mathbb{C}_{sys})$$

Where  $c_{sub}$ , and  $c_{sys}$  are the specific value of credibility variables respectively.

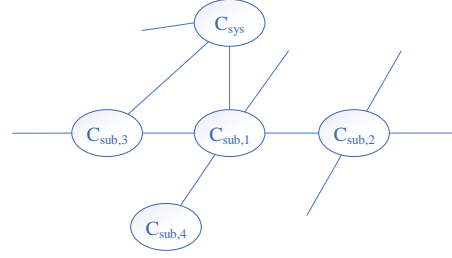


Figure 1. Credibility PMRF model

In the credibility PMRF  $\mathcal{H}$ , the unary potential function  $\phi$  characterizes the prior information of the credibility of simulation sub-systems and can be defined as follows:

$$\phi_{\alpha}(c_{sub,\alpha}) = \exp(-E_{\alpha}(c_{sub,\alpha})) \quad (3)$$

Where  $E$  is the energy function. Assuming the prior distribution of  $c_{sub,i}$  is Gaussian distribution,  $\phi_{\alpha}(c_{sub,\alpha})$  follows Gaussian distribution and  $E_{\alpha}(c_{sub,\alpha})$  can be computed as

$$E_{\alpha}(c_{sub,\alpha}) = -\log \left( \frac{1}{\sqrt{2\pi}\sigma_{\alpha}} \exp \left( -\frac{(c_{sub,\alpha} - \mu_i)^2}{2\sigma_{\alpha}^2} \right) \right) \quad (4)$$

In the complex simulation system, when the simulation sub-systems have some kind of connection relationships, the credibility of them have a high probability of being close. Hence, we prefer neighboring nodes in the credibility PMRF to have similar credibility. The energy functions of the pairwise potential function  $\varphi$  and  $\psi$  are defined as the distance metric which is the form of truncated p-norm, i.e.,

$$\begin{aligned} \varphi_{\beta}(c_{sub,\beta}, c_{sys}) &= \exp(-\omega_{\varphi} E_{\beta}(c_{sub,\beta}, c_{sys})) \\ E_{\beta}(c_{sub,\beta}, c_{sys}) &= \max(|c_{sub,\beta} - c_{sys}|_p, \delta_{\varphi}) \end{aligned} \quad (5)$$

$$\begin{aligned} \psi_{\eta\gamma}(c_{sub,\eta}, c_{sub,\gamma}) &= \exp(-\omega_{\psi} E_{\eta\gamma}(c_{sub,\eta}, c_{sub,\gamma})) \\ E_{\eta\gamma}(c_{sub,\eta}, c_{sub,\gamma}) &= \max(|c_{sub,\eta} - c_{sub,\gamma}|_p, \delta_{\psi}) \end{aligned} \quad (6)$$

Where  $\delta_{\varphi}$  and  $\delta_{\psi}$  are the lower bounds.

### 3.3. Credibility inference algorithm

According to the above factorization of  $\tilde{P}(\mathbb{C}_{sub}, \mathbb{C}_{sys})$ ,  $\tilde{P}(\mathbb{C}_{sub}, \mathbb{C}_{sys})$  can be rewritten as

$$\begin{aligned} \tilde{P}(\mathbb{C}_{sub}, \mathbb{C}_{sys}) &= \exp \left( -\sum_{\alpha} E_{\alpha}(c_{sub,\alpha}) - \sum_{\beta} \omega_{\varphi} E_{\beta}(c_{sub,\beta}, c_{sys}) \right. \\ &\quad \left. - \sum_{(\eta,\gamma) \in \mathcal{E}} \omega_{\psi} E_{\eta\gamma}(c_{sub,\eta}, c_{sub,\gamma}) \right) \end{aligned} \quad (7)$$

In order to infer the credibility of simulation sub-systems, we aim to compute:

$$\begin{aligned} \mathcal{C}^* &= \operatorname{argmax} P(\mathbb{C}_{sub}, \mathbb{C}_{sys} = \zeta) \\ &\propto \operatorname{argmax} \tilde{P}(\mathbb{C}_{sub}, \mathbb{C}_{sys} = \zeta) \end{aligned} \quad (8)$$

Thus:

$$\begin{aligned}
 & c^* \\
 &= \operatorname{argmin} \left( \sum_{\alpha} E_{\alpha}(c_{sub,\alpha}) + \sum_{\beta} \omega_{\varphi} E_{\beta}(c_{sub,\beta}, \zeta) \right. \\
 & \left. + \sum_{(\eta,\gamma) \in \mathcal{E}} \omega_{\psi} E_{\eta\gamma}(c_{sub,\eta}, c_{sub,\gamma}) \right) \quad (9.)
 \end{aligned}$$

We formulate the credibility optimization problem as a probabilistic inference problem which is also NP-hard. Nonetheless, the probabilistic inference problem can be tackled by using the inference algorithms of PMRF. Inference algorithms can be divided into two kinds: (1) exact inference algorithms, which compute posterior marginal probabilities exactly by systematically exploiting the graphical structure, and (2) approximate inference algorithms, which approximate posterior marginal probabilities by exploiting the numerical and the graph-theoretic properties of MRFs and reconstructing their joint distribution function (Jordan 2003). Exact inference algorithms can be performed efficiently for many MRFs. However, the computational complexity is exponential when the MRFs have large tree-width, and exact inference in these circumstances is intractable or even infeasible. Approximate inference algorithms construct the joint distribution function into a simpler distribution form which allows for inference and can be inferred using a variety of different methods. In addition, there are several different inference algorithms that are applicable to the same Markov networks and we can combine inference algorithms together to achieve perfect performance in practice. Inference algorithms commonly used are shown in Table 1.

**Table 1.** Common inference algorithms of PMRF.

Types of inference algorithms	Inference algorithms
Exact inference algorithms	Variable elimination (VE) algorithm; Sum-product algorithm; Clique tree algorithm
Approximate inference algorithms	Belief propagation (BP); Loopy belief propagation (LBP); Iterated conditional model (ICM); Mean field (MF); Sampling algorithms

The number of nodes and edges, variables' value range, and other reasons make the inference of MRFs complex and computationally expensive. Different inference algorithms have different inference precision and calculation speed for MRFs, which makes it difficult to choose an appropriate inference algorithm. Thus, it is necessary to analyze the inference results and performance of each inference algorithm before determining the inference algorithm.

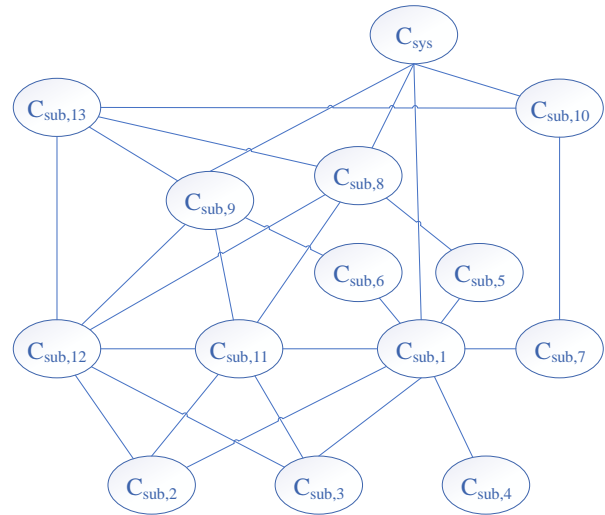
#### 4. Experiments

Taking the air defense combat complex simulation system as an example, we construct its credibility

PMRF and infer the credibility of each simulation sub-system. We use different inference algorithms to do inference and analyze the inference results, i.e., allocation results, under different circumstances.

The air defense combat complex simulation system is a complex simulation system featured with complicated interactions and varied system structure (Chen et al., 2012). The air defense combat complex simulation system consists of 13 simulation sub-systems, namely the command and control simulation sub-system, detection radar simulation sub-system, tracking radar simulation sub-system, fighter simulation sub-system, early warning aircraft simulation sub-system, missile launching vehicle simulation sub-system, artillery vehicle simulation sub-system, air-to-air missile simulation sub-system, surface-to-air missile simulation sub-system, artillery simulation sub-system, enemy reconnaissance plane simulation sub-system, enemy fighter plane simulation sub-system, air-to-ground missile simulation sub-system (Stary and Farlik, 2019). Let the credibility of the air defense combat complex simulation system is  $c_{sys}$  and the credibility of simulation sub-systems is  $c_{sub} = \{c_{sub,1}, c_{sub,2}, \dots, c_{sub,13}\}$ . According to the relationships between each sub-system, we construct the credibility PMRF  $\mathcal{G}$  of the air defense combat complex simulation system, which illustrates in Figure 2. Then,  $\tilde{P}(c_{sub}, c_{sys})$  of  $\mathcal{G}$  is

$$\begin{aligned}
 & \tilde{P}(c_{sub}, c_{sys}) \\
 &= \exp \left( \sum_{\alpha} E_{\alpha}(c_{sub,\alpha}) + \sum_{\beta} \omega_{\varphi} \max(|c_{sub,\beta} - c_{sys}|, \delta_{\varphi}) + \right. \\
 & \left. \sum_{(\eta,\gamma) \in \mathcal{E}} \omega_{\psi} \max(|c_{sub,\eta} - c_{sub,\gamma}|, \delta_{\psi}) \right) \quad (10.)
 \end{aligned}$$



**Figure 2.** Air defense combat simulation system credibility PMRF

Suppose the credibility takes values from  $\{0, 0.01, 0.02, \dots, 0.99, 1\}$  and the credibility requirement of the air defense combat complex simulation system is 0.8, i.e.,  $c_{sys} = 0.8$ . Besides, the prior distribution of each

node in  $\mathcal{G}$  is shown in Table 2. The credibility inference results are affected by the parameters  $\{\delta_\varphi, \delta_\psi, \omega_\varphi, \omega_\psi\}$  and inference algorithms. Thus, we analyze these influence factors in the following examples.

Firstly, we investigate the impact of  $\delta_\varphi$  and  $\delta_\psi$  on credibility inference results using the VE algorithm. Let  $\omega_\varphi = \omega_\psi = 1$  and the inference results are shown in Table 3 when  $\delta_\varphi$  and  $\delta_\psi$  take different values. We can see that the inference results are the same when  $\delta_\psi$  has the same value. Meanwhile, the larger  $\delta_\psi$  is, the closer the inference results are to the value with the highest prior probability. Therefore,  $\delta_\psi$  has a large effect on the inference results, making them tend to the values with a larger prior probability, while  $\delta_\varphi$  has a weak effect on the results.

Secondly, to study the effect of the parameter  $\omega_\varphi$  and  $\omega_\psi$ , let  $\delta_\varphi = \delta_\psi = 0.06$  and the inference results using the VE algorithm are summarized in Table 4. The parameter  $\omega_\varphi$  and  $\omega_\psi$  reflect the degree of influence of the relationships between neighbors on the credibility PMRF. That is, the parameters affect the weight of the pairwise potential in  $\tilde{P}(\mathcal{C}_{sub}, \mathcal{C}_{sys})$ , which makes the neighbors in the credibility PMRF tend to have the same credibility. Therefore, as illustrated in Table 4, the credibility of each simulation sub-system tends to be the same when  $\omega_\varphi$  and  $\omega_\psi$  are large enough, which

indicates that the parameter values should be selected appropriately.

Finally, let  $\omega_\varphi = \omega_\psi = 1$  and  $\delta_\varphi = \delta_\psi = 0.06$ . We infer the approximate credibility values of each simulation sub-system using LBP, ICM, and MF, as shown in Table 5. Since the VE algorithm can generate exact inference results, we can see that the inference performance of other approximate reasoning algorithms is MF, LBP, and ICM in order from best to worst. And even for LBP, the inference results are far from the correct results. Although these approximation algorithms cannot obtain exact inference results, their inference time is much shorter than that of the VE algorithm.

The parameters,  $\{\delta_\varphi, \delta_\psi, \omega_\varphi, \omega_\psi\}$ , have different effects on the final inference results, so it is necessary to choose appropriate values for each parameter according to the characteristics and credibility requirements of the complex simulation system. Furthermore, exact inference algorithms usually generate exact inference results but cost a long calculation time, while approximate inference algorithms are complete opposite. Hence, the inference algorithm should be chosen as a compromise and new inference algorithms which are suitable for the credibility allocation of simulation sub-systems need to be studied.

**Table 2.** The prior distribution of each node

Nodes	$\mathcal{C}_{sub,1}$	$\mathcal{C}_{sub,2}$	$\mathcal{C}_{sub,3}$	$\mathcal{C}_{sub,4}$	$\mathcal{C}_{sub,5}$	$\mathcal{C}_{sub,6}$	$\mathcal{C}_{sub,7}$	$\mathcal{C}_{sub,8}$
Prior Distribution	$U(0.75,0.95)$	$N(0.65,0.14)$	$N(0.72,0.1)$	$N(0.7,0.05)$	$U(0.57,0.75)$	$U(0.6,0.9)$	$U(0.75,1)$	$N(0.74,0.08)$
Nodes	$\mathcal{C}_{sub,9}$	$\mathcal{C}_{sub,10}$	$\mathcal{C}_{sub,11}$	$\mathcal{C}_{sub,12}$	$\mathcal{C}_{sub,13}$			
Prior Distribution	$N(0.8,0.05)$	$N(0.84,0.04)$	$N(0.7,0.2)$	$N(0.6,0.2)$	$N(0.72,0.15)$			

**Table 3.** Inference results for different values of  $\delta_\varphi$  and  $\delta_\psi$

Nodes	$\mathcal{C}_{sub,1}$	$\mathcal{C}_{sub,2}$	$\mathcal{C}_{sub,3}$	$\mathcal{C}_{sub,4}$	$\mathcal{C}_{sub,5}$	$\mathcal{C}_{sub,6}$	$\mathcal{C}_{sub,7}$	$\mathcal{C}_{sub,8}$	$\mathcal{C}_{sub,9}$	$\mathcal{C}_{sub,10}$	$\mathcal{C}_{sub,11}$	$\mathcal{C}_{sub,12}$	$\mathcal{C}_{sub,13}$
$\delta_\varphi = \delta_\psi = 0$	0.75	0.7	0.73	0.7	0.74	0.75	0.75	0.74	0.79	0.84	0.75	0.7	0.74
$\delta_\varphi = \delta_\psi = 0.06$	0.75	0.68	0.72	0.7	0.71	0.74	0.79	0.73	0.79	0.84	0.73	0.68	0.74
$\delta_\varphi = \delta_\psi = 0.12$	0.75	0.65	0.72	0.7	0.63	0.68	0.75	0.73	0.8	0.84	0.7	0.64	0.72
$\delta_\varphi = 0.12, \delta_\psi = 0$	0.75	0.7	0.73	0.7	0.74	0.75	0.75	0.74	0.79	0.84	0.75	0.7	0.74
$\delta_\varphi = 0, \delta_\psi = 0.12$	0.75	0.65	0.72	0.7	0.63	0.68	0.75	0.73	0.8	0.84	0.7	0.64	0.72

**Table 4.** Inference results for different values of  $\omega_\varphi$  and  $\omega_\psi$

Nodes	$\mathcal{C}_{sub,1}$	$\mathcal{C}_{sub,2}$	$\mathcal{C}_{sub,3}$	$\mathcal{C}_{sub,4}$	$\mathcal{C}_{sub,5}$	$\mathcal{C}_{sub,6}$	$\mathcal{C}_{sub,7}$	$\mathcal{C}_{sub,8}$	$\mathcal{C}_{sub,9}$	$\mathcal{C}_{sub,10}$	$\mathcal{C}_{sub,11}$	$\mathcal{C}_{sub,12}$	$\mathcal{C}_{sub,13}$
$\omega_\varphi = \omega_\psi = 0$	0.75	0.65	0.72	0.7	0.69	0.74	0.78	0.74	0.8	0.84	0.7	0.61	0.72
$\omega_\varphi = \omega_\psi = 1$	0.75	0.68	0.72	0.7	0.71	0.74	0.79	0.73	0.79	0.84	0.73	0.68	0.74
$\omega_\varphi = \omega_\psi = 10$	0.75	0.69	0.72	0.7	0.7	0.72	0.76	0.7	0.77	0.82	0.71	0.71	0.76

**Table 5.** Inference results for different inference algorithms

Nodes	$\mathcal{C}_{sub,1}$	$\mathcal{C}_{sub,2}$	$\mathcal{C}_{sub,3}$	$\mathcal{C}_{sub,4}$	$\mathcal{C}_{sub,5}$	$\mathcal{C}_{sub,6}$	$\mathcal{C}_{sub,7}$	$\mathcal{C}_{sub,8}$	$\mathcal{C}_{sub,9}$	$\mathcal{C}_{sub,10}$	$\mathcal{C}_{sub,11}$	$\mathcal{C}_{sub,12}$	$\mathcal{C}_{sub,13}$
VE	0.75	0.68	0.72	0.7	0.71	0.74	0.79	0.73	0.79	0.84	0.73	0.68	0.74
LBP	0.69	0.65	0.71	0.7	0.63	0.63	0.75	0.63	0.63	0.69	0.69	0.64	0.69
ICM	0.63	0.65	0.66	0.69	0.57	0.6	0.6	0.54	0.54	0.54	0.6	0.6	0.6
MF	0.69	0.65	0.71	0.7	0.64	0.65	0.75	0.64	0.64	0.82	0.68	0.64	0.69

## 5. Conclusions

To solve the credibility allocation problem, we formulate it as a probabilistic inference problem and

propose a credibility PMRF in this paper. The credibility of the complex simulation system and simulation sub-systems are modeled as nodes and the credibility relationships between them are modeled as edges. Then, we analyze the allocation task based on the

credibility PMRF and list several common inference algorithms. In the experiments on the air defense combat complex simulation system, we construct its credibility PMRF and analyze the inference performance under different parameter values and inference algorithms. According to the experimental results, we can see that parameters reflect the proportion of unary potential functions and the pairwise potential functions in the joint probability distribution respectively. Although the proposed approach could obtain the credibility allocation results fastly and accurately, there are also several limitations. The prior distribution of each simulation sub-system in the credibility PMRF cannot be obtained in many cases. The input-output relationships between simulation sub-systems are complex, which is difficult to be clearly represented by the edges between neighbors in PMRFs. Besides, although both exact inference algorithms and approximate inference algorithms can generate credibility allocation results, they have their own advantages and disadvantages. In future work, we will devote our best efforts to digging deep into the structure of the complex simulation system to construct more accurate MRFs and explore more appropriate inference algorithms.

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